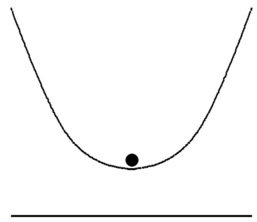
3.3 Types of inflation

Inflation requires a form of matter with $p < -\frac{1}{3}\rho$. This can be provided by positive vacuum energy density, or, more generally, by the potential energy density of a scalar field, both of which have $p = -\rho$.

In this section we will describe the types of inflation that emerge naturally from particle physics, and also the type that is required to produce an approximately scale-invariant spectrum of density perturbations.

3.3.1 Positive cosmological constant



This is the simplest type of inflation. The energy density of the universe is dominated by a positive cosmological constant, i.e. the positive energy density of our vacuum. The universe then expands exponentially

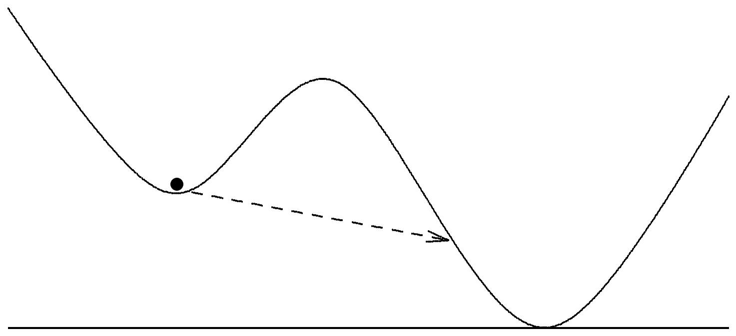
$$a \propto e^{Ht}, \quad H = \text{constant} \tag{135}$$

The energy density in any radiation or matter decays exponentially, as does the curvature

$$\rho_{\rm rad} \propto e^{-4Ht}, \quad \rho_{\rm mat} \propto e^{-3Ht}, \quad K \propto e^{-2Ht} \tag{136}$$

This type of inflation **never ends** and so cannot be the origin of our hot Big Bang universe.

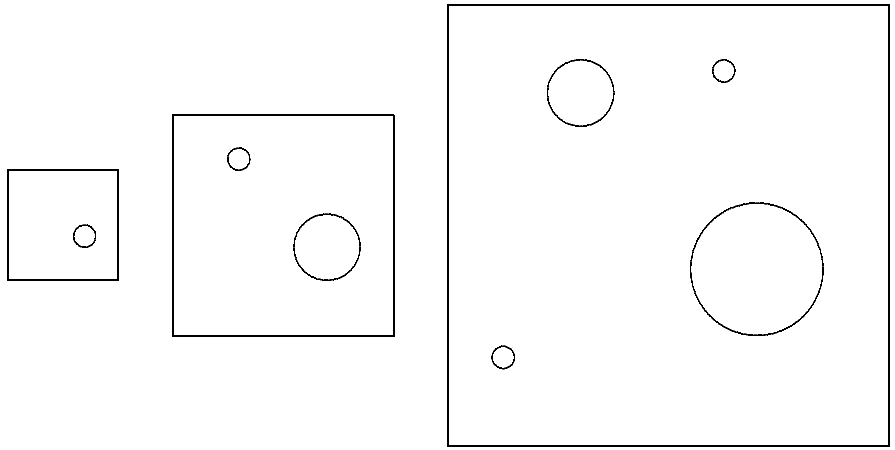
3.3.2 False vacuum inflation



Here the positive vacuum energy is that of a false vacuum so that inflation can end by quantum tunneling to the true vacuum. If the decay rate per unit volume $\Gamma \gtrsim H^4$, inflation will barely begin, so we will assume $\Gamma \lesssim H^4$.

Once the universe becomes trapped in the false vacuum, spacetime will tend to de Sitter space, which contains the exponentially large, flat, homogeneous and isotropic, spatial hypersurfaces that we want inflation to produce. However, because de Sitter space is not just spatially homogeneous, but is completely homogeneous, it has **no clock**, i.e. no unique choice of time-slicing, to distinguish these hypersurfaces from any others. The quantum tunneling is a random Poisson process and so also does not have a clock. Therefore, the end of inflation cannot be synchronized to occur on one of these exponentially large, flat, homogeneous and isotropic, spatial hypersurfaces, and so the achievements of the inflation are not preserved by the exit from inflation.

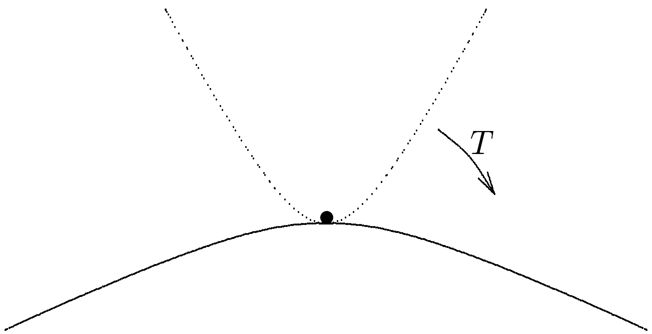
Indeed, if $\Gamma \lesssim H^4$, the inflation never ends completely because the volume of the universe which is inflating increases at a rate faster than it can be eaten up by the nucleating and expanding bubbles of true vacuum. One gets an **eternally inflating universe** continually nucleating bubbles of true vacuum. Each of these bubbles of true vacuum corresponds to



an infinite, negative curvature dominated, homogeneous and isotropic universe. Thus if a subsequent more phenomenologically acceptable inflation occurred within some of these bubble universes, one could have an eternally inflating universe filled with an infinite number of hot Big Bang bubble universes.

3.3.3 Thermal inflation

Here the finite temperature effective potential provides the false vacuum, and the temperature acts as a clock to synchronize the end of inflation.



Consider the finite temperature effective potential

$$V = V_0 + \frac{1}{2}\left(g^2T^2 - m^2\right)\phi^2 + \ldots \tag{137}$$

with $g \sim 1$. When

$$m \lesssim T \lesssim V_0^{1/4} \tag{138}$$

the scalar field ϕ is held at $\phi = 0$ by the finite temperature T, and the false vacuum energy density V_0 dominates the thermal energy density $\sim T^4$ and so drives an epoch of inflation.

The temperature T decreases during the inflation as

$$T \propto \frac{1}{a} \propto e^{-Ht}, \quad H = \sqrt{\frac{V_0}{3}} \tag{139}$$

and acts as a clock to synchronize the end of inflation. The inflation lasts for

$$N \sim \ln\left(\frac{T_{\rm initial}}{T_{\rm final}}\right) \sim \ln\left(\frac{V_0^{1/4}}{m}\right) \tag{140}$$

e-folds. To get a significant amount of inflation we require

$$m \ll V_0^{1/4} \tag{141}$$

For example, potentials of the form

$$V = V_0 - \frac{1}{2}m^2\phi^2 + \frac{\lambda^2}{M_{\rm Pl}^{2n}}\phi^{4+2n}, \quad n \geq 1 \tag{142}$$

which are common in supersymmetric theories, have their minimum at $\phi \equiv M \gg m$, and so in order to cancel the cosmological constant have $V_0 \sim m^2M^2 \gg m^4$. For $m \sim m_{\rm EW} \sim 10^{-16}$ and $\lambda \sim 1$ we get

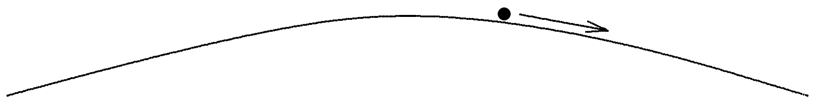
$$N \sim \frac{n}{2(n+1)}\ln\left(\frac{1}{m}\right) \sim 18\left(\frac{n}{n+1}\right) \tag{143}$$

Thus, even for $n = \infty$, which corresponds to $M = M_{\rm Pl}$, a single epoch of thermal inflation does **not** give **enough *e*-folds** of inflation to make the universe big, flat, etc.

Also, thermal inflation is **not** a **scale-invariant** process, the temperature falls like $T \propto 1/a$, and so can not produce a scale-invariant spectrum of density perturbations.

3.3.4 Rolling scalar field inflation

Here a rolling scalar field provides the clock that synchronizes the end of inflation.



Inflation requires $p < -\rho/3$, and for a homogeneous scalar field

$$\rho = \frac{1}{2}\dot{\phi}^2 + V \tag{144}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V \tag{145}$$

and so to get inflation we need

$$\dot{\phi}^2 < V \tag{146}$$

This is most simply achieved near a maximum of the potential

$$V = V_0 - \frac{1}{2}m^2\phi^2 + \ldots \tag{147}$$

The equation of motion for the scalar field is

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \tag{148}$$

The friction term $3H\dot{\phi}$ arises from the expansion of the universe which damps the kinetic energy.

If $\phi = 0$, or is sufficiently small, the dynamics will be dominated by quantum fluctuations. Using the results of Section 3.4, one can show that one gets **eternal inflation** in the neighborhood of $\phi = 0$ if $m^2 \leq 6V_0$.

Once ϕ escapes from the neighborhood of zero, which we crudely take to occur when $\phi \gtrsim H$, the classical motion will dominate. While ϕ is still near the top of the potential, $\phi \ll V_0^{1/2}/m$, we have $H \simeq \sqrt{V_0/3}$, and so can solve the equation of motion to give

$$\phi \propto a^\alpha \tag{149}$$

where

$$\alpha = \frac{3}{2}\left(\sqrt{1 + \frac{4m^2}{3V_0}} - 1\right) \tag{150}$$

Inflation will end when $\phi \sim V_0^{1/2}/m$. Therefore the number of e-folds of this classical rolling inflation is

$$N \sim \frac{1}{\alpha}\ln\left(\frac{\phi_{\mathrm{end}}}{\phi_{\mathrm{initial}}}\right) \sim \frac{1}{\alpha}\ln\left(\frac{1}{m}\right) \tag{151}$$

For $m \sim m_{\mathrm{EW}} \sim 10^{-16}$ this gives

$$N \sim \frac{37}{\alpha} \tag{152}$$

To obtain a significant amount of inflation we require

$$m \lesssim \frac{V_0^{1/2}}{M_{\mathrm{Pl}}} \tag{153}$$

which is a much stronger constraint than that required by thermal inflation ($m \ll V_0^{1/4}$).

From Eq. (149) we see that rolling scalar field inflation is **not scale-invariant**, unless $\alpha \ll 1$ which is the slow-roll limit to be discussed the next section, and so will not in general produce a scale-invariant spectrum of density perturbations.

3.3.5 Slow-roll inflation

This is the **scale-invariant** limit of rolling scalar field inflation.

Observations show that the density perturbations are approximately scale-invariant on the largest observable scales. This presumably means that the inflation that produced them should be an approximately scale-invariant process. This in turn means that, during inflation, physical quantities such as the potential and kinetic energy densities should remain approximately constant as the scale factor a increases. This is quantified by

$$\left|\frac{d\ln X}{d\ln a}\right| \ll 1 \tag{154}$$

for all relevant quantities X that do not depend explicitly on the scale factor a. For example

$$-\frac{d\ln H}{d\ln a} = -\frac{\dot{H}}{H^2} \ll 1 \tag{155}$$

and

$$\left|\frac{d\ln\dot{\phi}}{d\ln a}\right| = \left|\frac{\ddot{\phi}}{H\dot{\phi}}\right| \ll 1 \tag{156}$$

Eq. (155) is usually valid in other types of inflation as well. It gives

$$3H^2 \simeq V \tag{157}$$

Using Eq. (156), the equation of motion $\ddot{\phi} + 3H\dot{\phi} + V' = 0$ is approximated by the characteristic, friction dominated, **slow-roll equation of motion**

$$3H\dot{\phi} + V' = 0 \tag{158}$$

Ignoring transients, in terms of the potential Eqs. (155) and (156) translate to

$$\left(\frac{V'}{V}\right)^2 \ll \frac{1}{M_{\rm Pl}^2} \tag{159}$$

and

$$\left|\frac{V''}{V}\right| \ll \frac{1}{M_{\rm Pl}^2} \tag{160}$$

The first suggests we should be near a maximum, or other extremum, of the potential. The second is non-trivial, and in fact provides one of the most serious obstacles to building a model of slow-roll inflation. To get a sense of why this is so, note that if you try to build a model of inflation in an effective field theory that neglects gravitational strength interactions, then you are implicitly setting $M_{\rm Pl} = \infty$. Clearly one cannot achieve Eq. (160) in this context.

If instead one works in supergravity, then one can show that if the inflationary potential energy density is dominated by the F-term then

$$\frac{V''}{V} = \frac{1}{M_{\rm Pl}^2} + \text{model dependent terms} \tag{161}$$

Therefore, to build a model of slow-roll inflation one must be able to **control the gravitational strength interactions**.

References

1. E. D. Stewart, Physical Review D51 (1995) 6847-6853.