1 Overview

1.1 Units

Throughout these lectures I will use natural units [1]

$$c = 8\pi G = \hbar = 1 \tag{1}$$

Occasionally, I will reintroduce some units for clarity, for emphasis, or to follow standard conventions. I define the Planck mass as

$$M_{\mathrm{Pl}} \equiv \left(\frac{\hbar c}{8\pi G}\right)^{1/2} = 4\,\mu\mathrm{g} \tag{2}$$

The Planck length and time are defined similarly and are extremely small.

The standard units of high energy physics are to set $c = \hbar = 1$ and use the giga electron volt or GeV as the remaining dimensionful parameter. In these units, the mass of an electron is

$$m_{\mathrm{e}} = 5 \times 10^{-4}\,\mathrm{GeV} \tag{3}$$

The mass of a proton or neutron is

$$m_{\mathrm{p}} \simeq m_{\mathrm{n}} = 0.9\,\mathrm{GeV} \tag{4}$$

The scale of electro-weak symmetry breaking is

$$m_{\mathrm{EW}} \sim 10^{2}\,\mathrm{GeV} \tag{5}$$

The Planck mass is

$$M_{\mathrm{Pl}} = 2.436 \times 10^{18}\,\mathrm{GeV} = 9 \times 10^{4}\,\mathrm{kcal} \tag{6}$$

[1] Note that many authors instead choose to set $G = 1$.

The standard units of the late universe are megaparsecs or Mpc for length

$$\mathrm{pc} = 3.3\,c\,\mathrm{yr} \tag{7}$$

$$\mathrm{Mpc} = 1.6 \times 10^{38}\,\mathrm{GeV}^{-1} = 3.8 \times 10^{56} \tag{8}$$

$\mathrm{km\,s^{-1}}$ for velocity

$$\mathrm{km\,s^{-1}} = 3.3 \times 10^{-6} \tag{9}$$

years for time

$$\mathrm{yr} = 4.8 \times 10^{31}\,\mathrm{GeV}^{-1} = 1.2 \times 10^{50} \tag{10}$$

and solar masses for mass

$$M_{\odot} = 1.1 \times 10^{57}\,\mathrm{GeV} = 4.6 \times 10^{38} \tag{11}$$

1.2 Basic observed properties of the universe

1. The universe is **old**.
The oldest objects in the universe whose age has been estimated are about 10^{10} years old.

$$T_0 \gtrsim 10^{10}\,\mathrm{yr} \sim 10^{60} \tag{12}$$

2. The universe is **big**.
No observable boundary or periodicity.

$$L_0 \gtrsim 10^{10}\, c\,\mathrm{yr} \sim 10^{60} \tag{13}$$

3. There is a **lot of matter** in the universe.

$$M_0 \gtrsim 10^{21} M_\odot \sim 10^{60} \tag{14}$$

4. The universe is approximately **homogeneous** and **isotropic** on the largest observable scales.

5. The observable universe is **expanding**.
The expansion rate is given by the Hubble parameter H.
The current value of the Hubble parameter is

$$H_0 \simeq 65\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{15}$$
$$\sim \frac{1}{10^{10}\,\mathrm{yr}} \sim 10^{-42}\,\mathrm{GeV} \sim 10^{-60} \tag{16}$$

6. The universe is **flat**.
There is no observable large scale spatial curvature.

7. The universe is not exactly homogeneous and isotropic. **Density perturbations** have an amplitude $\sim 10^{-5}$ and an approximately **scale-invariant** spectrum on the largest observable scales.

8. The matter content of the universe is dominated by **cold dark matter** with about 15% ordinary matter.

$$\rho_{\mathrm{CDM}} \simeq 6\rho_{\mathrm{B}} \tag{17}$$

9. The **baryon** [2] to cosmic microwave background **photon ratio** is about 5×10^{-10}.

$$n_\gamma \simeq 2 \times 10^9 n_{\mathrm{B}} \tag{18}$$

10. The current temperature of the **cosmic microwave background radiation** is

$$T_{\gamma 0} = 2.73\,\mathrm{K} = 2.35 \times 10^{-4}\,\mathrm{eV} \tag{19}$$

which corresponds to a photon number density

$$n_{\gamma 0} = 0.41\,\mathrm{mm}^{-3} \tag{20}$$

[2] A baryon is a particle made of three quarks, for example, a proton or neutron. At energies below the electro-weak scale, the number of baryons is conserved to a very good approximation.