Problem Set 2 - Metric and matter

1. The metric of a (negative) curvature dominated universe is
$$ds^2 = dt^2 - t^2 \left(dr^2 + \sinh^2 r \, d\Omega^2 \right) \tag{1}$$
where
$$d\Omega^2 = d\theta^2 + \sin^2 \theta \, d\phi^2 \tag{2}$$
is the metric on a unit two-sphere. Make the change of coordinates
$$t' = t \cosh r \tag{3}$$
$$r' = t \sinh r \tag{4}$$
What do you get? Explain.

2. Derive the equation of motion of a scalar field in a homogeneous and isotropic universe. Show that a homogeneous rapidly oscillating scalar field with potential $V = \frac{1}{2} m^2 \phi^2$ and mass $m \gg H$ has zero pressure.

3. Can a scalar field be described by a perfect fluid?