

Homework 1 - Curvature tensor

Q1.1. Calculate

$$(\nabla_{\mathbf{a}}\nabla_{\mathbf{b}} - \nabla_{\mathbf{b}}\nabla_{\mathbf{a}})v^{\mathbf{c}} \quad (\text{Q1.1.1})$$

in two different ways, and hence show that

$$R_{\mathbf{abcd}} = -R_{\mathbf{abdc}} \quad (\text{Q1.1.2})$$

Find all possible distinct contractions of the metric with the curvature tensor.

Q1.2. Use the curvature tensor identity

$$R_{\mathbf{abc}}{}^{\mathbf{d}} + R_{\mathbf{bca}}{}^{\mathbf{d}} + R_{\mathbf{cab}}{}^{\mathbf{d}} = 0 \quad (\text{Q1.2.1})$$

to show that

$$R_{\mathbf{abcd}} = R_{\mathbf{cdab}} \quad (\text{Q1.2.2})$$