Homework 2 - Bases and coordinates

Q2.1. Show that

(a)
$$v^\alpha = e^\alpha_{\mathbf{a}} v^{\mathbf{a}} \tag{Q2.1.1}$$

(b)
$$\omega_{\mathbf{a}} v^{\mathbf{a}} = \omega_\alpha v^\alpha \tag{Q2.1.2}$$

(c)
$$e^\alpha_{\mathbf{a}} e^{\mathbf{b}}_\alpha = \delta^{\mathbf{b}}_{\mathbf{a}} \tag{Q2.1.3}$$

and explain the meaning of all the terms.

Q2.2. Let $e^{\mathbf{a}}_r$ and $e^{\mathbf{a}}_\theta$ be the coordinate basis vectors associated with polar coordinates in two dimensional Euclidean space, and $e^{\mathbf{a}}_{\hat{r}}$ and $e^{\mathbf{a}}_{\hat{\theta}}$ be the orthonormal basis vectors proportional to $e^{\mathbf{a}}_r$ and $e^{\mathbf{a}}_\theta$.

(a) Express the metric $g_{\mathbf{ab}}$ and inverse metric $g^{\mathbf{ab}}$ in terms of the coordinate and orthonormal bases.

(b) Express the coordinate basis vectors and covectors in terms of the orthonormal basis vectors and covectors.

(c) Draw simple diagrams illustrating $e^{\mathbf{a}}_r$, $e^{\mathbf{a}}_\theta$, $e^r_{\mathbf{a}}$, $e^\theta_{\mathbf{a}}$ $e^{\mathbf{a}}_{\hat{r}}$, $e^{\mathbf{a}}_{\hat{\theta}}$, $e^{\hat{r}}_{\mathbf{a}}$ and $e^{\hat{\theta}}_{\mathbf{a}}$.