Homework 3 - Christoffel symbols

Q3.1. Show that

$$\nabla_{\mathbf{a}} e^{\gamma}_{\mathbf{b}} = -\Gamma^{\gamma}_{\alpha\beta} e^{\alpha}_{\mathbf{a}} e^{\beta}_{\mathbf{b}} \tag{Q3.1.1}$$

and hence derive Eq. (1.2.28) and show that, for zero torsion and in a coordinate basis,

$$\Gamma^{\gamma}_{\alpha\beta} = \Gamma^{\gamma}_{\beta\alpha} \tag{Q3.1.2}$$

Q3.2. Derive Eq. (1.2.29).

Q3.3. Express the curvature tensor in terms of the Christoffel symbols.

Q3.4. Let $e^{\mathbf{a}}_{r}$ and $e^{\mathbf{a}}_{\theta}$ be the coordinate basis vectors associated with polar coordinates in two dimensional Euclidean space, and $e^{\mathbf{a}}_{\hat{r}}$ and $e^{\mathbf{a}}_{\hat{\theta}}$ be the orthonormal basis vectors proportional to $e^{\mathbf{a}}_{r}$ and $e^{\mathbf{a}}_{\theta}$.

(a) Calculate the $\Gamma^{\gamma}_{\alpha\beta}$ for the coordinate basis.

(b) Express the velocity and acceleration in terms of the coordinate and orthonormal bases.

(c) Write down the equation of a geodesic in the coordinate basis and check that the geodesics are straight lines.