# Homework 4 - Lie derivative

Q4.1. Show that

(a) the Lie derivative is independent of the metric,

(b) the Lie derivative, with respect to a vector field $u^{\mathbf{a}}$, acting on a covector field $\omega_{\mathbf{a}}$ is

$$\mathcal{L}_u \omega_{\mathbf{a}} = u^{\mathbf{b}} \nabla_{\mathbf{b}} \omega_{\mathbf{a}} + \left( \nabla_{\mathbf{a}} u^{\mathbf{b}} \right) \omega_{\mathbf{b}} \tag{Q4.1.1}$$

(c) Derive Eq. (1.2.9).

(d) Show that a coordinate basis vector $e^{\mathbf{a}}_{\alpha}$ is a Killing vector if and only if

$$\nabla_{\alpha} g_{\beta\gamma} = 0 \tag{Q4.1.2}$$

for all $\beta, \gamma$, and explain the difference between $\nabla_{\alpha} g_{\beta\gamma}$ and $\nabla_{\mathbf{a}} g_{\mathbf{bc}}$.

(e) Show that a particle with momentum

$$p_{\mathbf{a}} = m g_{\mathbf{ab}} \frac{dx^{\mathbf{b}}}{dt} \tag{Q4.1.3}$$

and moving freely in a space with Killing vector field $\xi^{\mathbf{a}}$ has conserved quantity $\xi^{\mathbf{a}} p_{\mathbf{a}}$.

(f) By considering Cartesian and polar coordinates, determine the conserved quantities of a particle moving freely in two dimensional Euclidean space.