Homework 8 - Penrose diagrams

Q8.1. Draw the penrose diagram for Minkowski space.

A8.1. The Minkowski metric is

$$d\tau^2 = dt^2 - dr^2 - r^2\, d\Omega^2 \tag{A8.1.1}$$

Defining the null coordinates

$$u = (t + r)/\sqrt{2} \tag{A8.1.2}$$
$$v = (t - r)/\sqrt{2} \tag{A8.1.3}$$

gives

$$d\tau^2 = 2\, du\, dv - \frac{1}{2}(u - v)^2\, d\Omega^2 \tag{A8.1.4}$$

The coordinates

$$u' = \tan^{-1} u \tag{A8.1.5}$$
$$v' = \tan^{-1} v \tag{A8.1.6}$$

bring infinity in to finite values

$$u', v' \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \tag{A8.1.7}$$

and a conformal rescaling of the metric

$$\hat{g}_{\mathbf{ab}} = \cos^2 u' \cos^2 v' g_{\mathbf{ab}} \tag{A8.1.8}$$

gives

$$d\hat{\tau}^2 = 2\, du'\, dv' - \frac{1}{2}\sin^2(u' - v')\, d\Omega^2 \tag{A8.1.9}$$

and hence Figure A8.1.1.

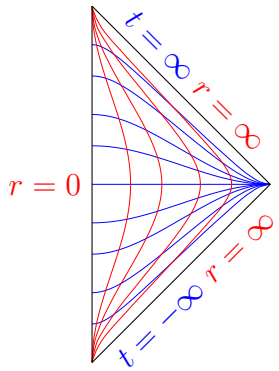

Figure A8.1.1: Penrose diagram for Minkowski spacetime.