

## Homework 11 - Expanding universe

Q11.1. Calculate the time  $t$  as a function of the scale factor  $a$  for a universe dominated by radiation

$$p_r = \frac{1}{3}\rho_r \quad (\text{Q11.1.1})$$

then matter

$$p_m = 0 \quad (\text{Q11.1.2})$$

then vacuum energy

$$p_v = -\rho_v \quad (\text{Q11.1.3})$$

including the transitions between these eras, and show that your answer reduces to

$$a_r \simeq \left(\frac{4\rho_r a^4}{3}\right)^{\frac{1}{4}} t^{\frac{1}{2}} \quad (\text{Q11.1.4})$$

$$a_m \simeq \left(\frac{3\rho_m a^3}{4}\right)^{\frac{1}{3}} t^{\frac{2}{3}} \quad (\text{Q11.1.5})$$

$$a_v \simeq \left(\frac{\rho_m a^3}{4\rho_v}\right) \exp\left(\sqrt{\frac{\rho_v}{3}} t\right) \quad (\text{Q11.1.6})$$

when the respective individual components dominate.