Homework 12 - Quantum fields in curved spacetime

Q12.1. Show that if

$$\varphi(\eta, \vec{x}) = \int \frac{d^3\underline{k}}{(2\pi)^{3/2}} \left[a_{\underline{k}}\, \varphi_k(\eta) + a^\dagger_{-\underline{k}}\, \varphi^*_k(\eta) \right] e^{i\underline{k}\cdot\vec{x}} \tag{Q12.1.1}$$

with φ_k normalised such that

$$\varphi_k \varphi^{*\prime}_k - \varphi'_k \varphi^*_k = i \tag{Q12.1.2}$$

then the commutation relations

$$[\varphi(\eta, \vec{x}), \varphi(\eta, \vec{y})] = 0 \tag{Q12.1.3}$$

$$[\varphi(\eta, \vec{x}), \varphi'(\eta, \vec{y})] = i\,\delta^3(\vec{x} - \vec{y}) \tag{Q12.1.4}$$

$$[\varphi'(\eta, \vec{x}), \varphi'(\eta, \vec{y})] = 0 \tag{Q12.1.5}$$

imply

$$\left[a_{\underline{k}}, a_{\underline{l}}\right] = 0 \quad , \quad \left[a_{\underline{k}}, a^\dagger_{\underline{l}}\right] = i\,\delta^3(\vec{x} - \vec{y}) \quad , \quad \left[a^\dagger_{\underline{k}}, a^\dagger_{\underline{l}}\right] = 0 \tag{Q12.1.6}$$