Homework 2 - Tensor algebra

Q2.1. Use diagrammatic methods to show that

$$\vec{a} \cdot (\underline{b} \wedge \underline{c}) = (\vec{a} \cdot \underline{b})\, \underline{c} - (\vec{a} \cdot \underline{c})\, \underline{b} \tag{Q2.1.1}$$

A2.1. Rescaling

$$\hat{\underline{b}} = \frac{\underline{b}}{\vec{a} \cdot \underline{b}} \tag{A2.1.1}$$

$$\hat{\underline{c}} = \frac{\underline{c}}{\vec{a} \cdot \underline{c}} \tag{A2.1.2}$$

then Figure A2.1.1 gives

$$\vec{a} \cdot \left(\hat{\underline{b}} \wedge \hat{\underline{c}}\right) = \hat{\underline{c}} - \hat{\underline{b}} \tag{A2.1.3}$$

and hence Eq. (Q2.1.1).

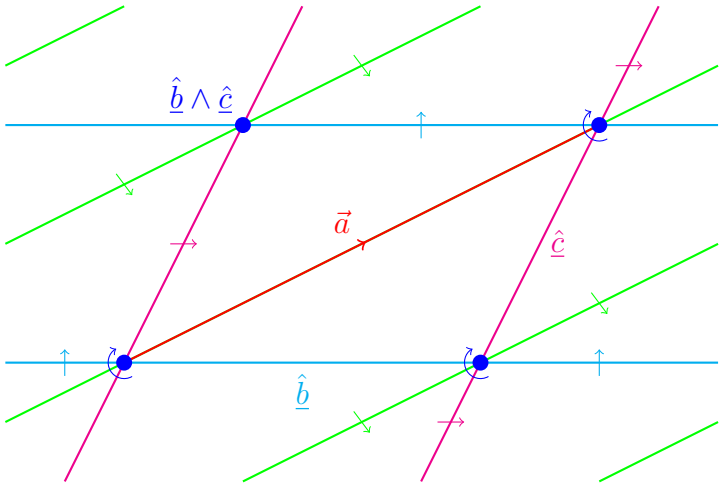

Figure A2.1.1: $\left[\vec{a} \cdot \left(\hat{\underline{b}} \wedge \hat{\underline{c}}\right)\right] = \left(\hat{\underline{c}} - \hat{\underline{b}}\right)$

Q2.2. Using Eq. (1.1.22), show that

(a)

$$(\underline{\omega} \wedge \underline{\sigma}) \cdot (\vec{u} \wedge \vec{v}) = (\underline{\omega} \cdot \vec{u})\, (\underline{\sigma} \cdot \vec{v}) - (\underline{\omega} \cdot \vec{v})\, (\underline{\sigma} \cdot \vec{u}) \tag{Q2.2.1}$$

(b)

$$\underline{\sigma} \cdot (\vec{u} \wedge \vec{v} \wedge \vec{w}) = (\underline{\sigma} \cdot \vec{u})\, \vec{v} \wedge \vec{w} + (\underline{\sigma} \cdot \vec{v})\, \vec{w} \wedge \vec{u} + (\underline{\sigma} \cdot \vec{w})\, \vec{u} \wedge \vec{v} \tag{Q2.2.2}$$

A2.2. (a)

$$(\underline{\omega} \wedge \underline{\sigma}) \cdot (\vec{u} \wedge \vec{v}) = [(\underline{\omega} \wedge \underline{\sigma}) \cdot \vec{v}] \cdot \vec{u} \tag{A2.2.1}$$

$$= [\underline{\omega}\, (\underline{\sigma} \cdot \vec{v}) - (\underline{\omega} \cdot \vec{v})\, \underline{\sigma}] \cdot \vec{u} \tag{A2.2.2}$$

$$= (\underline{\omega} \cdot \vec{u})\, (\underline{\sigma} \cdot \vec{v}) - (\underline{\omega} \cdot \vec{v})\, (\underline{\sigma} \cdot \vec{u}) \tag{A2.2.3}$$

(b)

$$\begin{aligned}
\underline{\sigma} \cdot (\vec{u} \wedge \vec{v} \wedge \vec{w}) &= (\underline{\sigma} \cdot \vec{u})\, \vec{v} \wedge \vec{w} + [\underline{\sigma} \cdot (\vec{v} \wedge \vec{w})] \wedge \vec{u} && \text{(A2.2.4)} \\
&= (\underline{\sigma} \cdot \vec{u})\, \vec{v} \wedge \vec{w} + [(\underline{\sigma} \cdot \vec{v})\, \vec{w} - (\underline{\sigma} \cdot \vec{w})\, \vec{v}] \wedge \vec{u} && \text{(A2.2.5)} \\
&= (\underline{\sigma} \cdot \vec{u})\, \vec{v} \wedge \vec{w} + (\underline{\sigma} \cdot \vec{v})\, \vec{w} \wedge \vec{u} + (\underline{\sigma} \cdot \vec{w})\, \vec{u} \wedge \vec{v} && \text{(A2.2.6)}
\end{aligned}$$