Homework 4 - Densities and volumes

Q4.1. Explain the conceptual difference between $\underset{\equiv}{\rho}$ and $\rho = \star\underset{\equiv}{\rho}$ in three dimensions.

Q4.2. Show that

(a)
$$\boldsymbol{\epsilon}^{-1} \cdot (\vec{v} \cdot \boldsymbol{\epsilon}) = \vec{v} \tag{Q4.2.1}$$

(b)
$$\boldsymbol{\epsilon}^{-1} \cdot [(\vec{u} \wedge \vec{v}) \cdot \boldsymbol{\epsilon}] = \vec{u} \wedge \vec{v} \tag{Q4.2.2}$$

Q4.3. Show that for an m-form $\boldsymbol{\omega}$ and an n-vector $\boldsymbol{v}$ with $m \leq n$

(a)
$$(\star\boldsymbol{\omega}) \cdot (\star\boldsymbol{v}) = \boldsymbol{\omega} \cdot \boldsymbol{v} \tag{Q4.3.1}$$

(b)
$$\star^{-1} (\boldsymbol{\omega} \wedge \star\boldsymbol{v}) = \boldsymbol{v} \cdot \boldsymbol{\omega} \tag{Q4.3.2}$$

Q4.4. Show that
$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0 \tag{Q4.4.1}$$
for any multivector $\boldsymbol{v}$. What traditional vector calculus results does this correspond to?