

Homework 6 - Abstract index notation

Q6.1. Express Eqs (1.4.3), (1.4.6) and (1.4.8) in abstract index notation.

Q6.2. Express Eqs. (1.5.2) and (1.5.3) in abstract index notation and show that

$$(a) \quad v^\alpha = e_{\mathbf{a}}^\alpha v^{\mathbf{a}} \quad (\text{Q6.2.1})$$

$$(b) \quad \omega_{\mathbf{a}} v^{\mathbf{a}} = \omega_\alpha v^\alpha \quad (\text{Q6.2.2})$$

$$(c) \quad e_{\mathbf{a}}^\alpha e_\alpha^{\mathbf{b}} = \delta_{\mathbf{a}}^{\mathbf{b}} \quad (\text{Q6.2.3})$$

explaining the meaning of all terms.

Q6.3. Using abstract index notation, show that

$$(a) \quad \vec{v} \cdot (\underline{\omega} \wedge \underline{\sigma}) = (\vec{v} \cdot \underline{\omega}) \underline{\sigma} + (\vec{v} \cdot \underline{\sigma}) \wedge \underline{\omega} \quad (\text{Q6.3.1})$$

$$(b) \quad \vec{v} \cdot (\underline{\omega} \wedge \underline{\sigma}) = \left(\vec{v} \cdot \underline{\sigma} \right) \cdot \underline{\omega} \quad (\text{Q6.3.2})$$