

## Homework 8 - Metric and volume form

Q8.1. Use abstract index notation to show that

$$\star \mathbf{v} = \mathbf{v} \cdot \boldsymbol{\epsilon} \quad \Rightarrow \quad \mathbf{v} = \boldsymbol{\epsilon}^{-1} \cdot \star \mathbf{v} \quad (\text{Q8.1.1})$$

Q8.2. Show that for an  $n$ -form  $\boldsymbol{\omega}$  in an  $N$ -dimensional space

$$\star^{-1} \boldsymbol{\omega} = (-1)^{n(N-n)} \text{sgn}(g) \star \boldsymbol{\omega} \quad (\text{Q8.2.1})$$

Q8.3. (a) Express Maxwell's equations in terms of  $\underline{E}$  and  $\underline{B}$ .

(b) Use

$$\underline{G} = \star \underline{F} - \underline{N} \quad (\text{Q8.3.1})$$

to express Maxwell's equations in terms of  $\underline{F}$

(c) Use

$$\star^{(4)} (\underline{e}^t \wedge \underline{E}) = - \star^{(3)} \underline{E} \quad (\text{Q8.3.2})$$

and

$$\star^{(4)} \underline{B} = \underline{e}^t \wedge \star^{(3)} \underline{B} \quad (\text{Q8.3.3})$$

to show that

$$\underline{N} = -\underline{e}^t \wedge \underline{M} + \underline{P} \quad (\text{Q8.3.4})$$

(d) Give the topological and physical meanings of  $\underline{\nabla} \wedge \underline{N}$  from both the four dimensional and three plus one dimensional points of view.