

Homework 10 - Christoffel symbols

Q10.1. Show that

$$\nabla_{\mathbf{a}} e_{\mathbf{b}}^{\gamma} = -\Gamma_{\alpha\beta}^{\gamma} e_{\mathbf{a}}^{\alpha} e_{\mathbf{b}}^{\beta} \quad (\text{Q10.1.1})$$

and hence derive Eq. (2.2.9) and show that, for zero torsion and in a coordinate basis,

$$\Gamma_{\alpha\beta}^{\gamma} = \Gamma_{\beta\alpha}^{\gamma} \quad (\text{Q10.1.2})$$

Q10.2. Derive Eq. (2.2.10).

Q10.3. Express the curvature tensor in terms of the Christoffel symbols.

Q10.4. Let $e_r^{\mathbf{a}}$ and $e_{\theta}^{\mathbf{a}}$ be the coordinate basis vectors associated with polar coordinates in two dimensional Euclidean space, and $e_r^{\hat{\mathbf{a}}}$ and $e_{\theta}^{\hat{\mathbf{a}}}$ be the orthonormal basis vectors proportional to $e_r^{\mathbf{a}}$ and $e_{\theta}^{\mathbf{a}}$.

- (a) Calculate the $\Gamma_{\alpha\beta}^{\gamma}$ for the coordinate basis.
- (b) Express the velocity and acceleration in terms of the coordinate and orthonormal bases.
- (c) Write down the equation of a geodesic in the coordinate basis and check that the geodesics are straight lines.