Homework 11 - Calculus of variations

Q11.1. The action functional

$$S[x(t)] = \int_{t_\mathrm{i}}^{t_\mathrm{f}} L(x, \dot{x}, t)\, dt \tag{Q11.1.1}$$

can be varied either covariantly

$$\frac{\delta S}{\delta x^{\mathbf{a}}} = \frac{\partial L}{\partial x^{\mathbf{a}}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^{\mathbf{a}}}\right) \tag{Q11.1.2}$$

or with respect to the coordinate paths

$$\frac{\delta S}{\delta x^{\alpha}} = \frac{\partial L}{\partial x^{\alpha}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^{\alpha}}\right) \tag{Q11.1.3}$$

(a) Show that

$$\frac{\delta S}{\delta x^{\mathbf{a}}} = \frac{\delta S}{\delta x^{\alpha}} e^{\alpha}_{\mathbf{a}} \tag{Q11.1.4}$$

(b) Evaluate Eqs. (Q11.1.2) and (Q11.1.3) for

$$L = \frac{1}{2} m g_{\mathbf{ab}} \dot{x}^{\mathbf{a}} \dot{x}^{\mathbf{b}} - V(x) \tag{Q11.1.5}$$

and show that they are equivalent.