

Problem Set 2 - Metric and matter

1. The metric of a (negative) curvature dominated universe is

$$ds^2 = dt^2 - t^2 (dr^2 + \sinh^2 r d\Omega^2) \quad (1)$$

where

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \quad (2)$$

is the metric on a unit two-sphere. Make the change of coordinates

$$t' = t \cosh r \quad (3)$$

$$r' = t \sinh r \quad (4)$$

What do you get? Explain.

2. Derive the equation of motion of a scalar field in a homogeneous and isotropic universe. Show that a homogeneous rapidly oscillating scalar field with potential $V = \frac{1}{2}m^2\phi^2$ and mass $m \gg H$ has zero pressure.
3. Can a scalar field be described by a perfect fluid?