

A Minimal Supersymmetric Cosmological Model

Ewan Stewart

KAIST

Prometeo I: LHC Physics and Cosmology
4 March 2009

Dept. of Theoretical Physics, University of Valencia

Donghui Jeong, Kenji Kadota, Wan-II Park, EDS	hep-ph/0406136
Gary N Felder, Hyunbyuk Kim, Wan-II Park, EDS	hep-ph/0703275
Richard Easterer, John T Giblin, Eugene A Lim, Wan-II Park, EDS	arXiv:0801.4197
Seongcheol Kim, Wan-II Park, EDS	arXiv:0807.3607

A Minimal Supersymmetric Cosmological Model

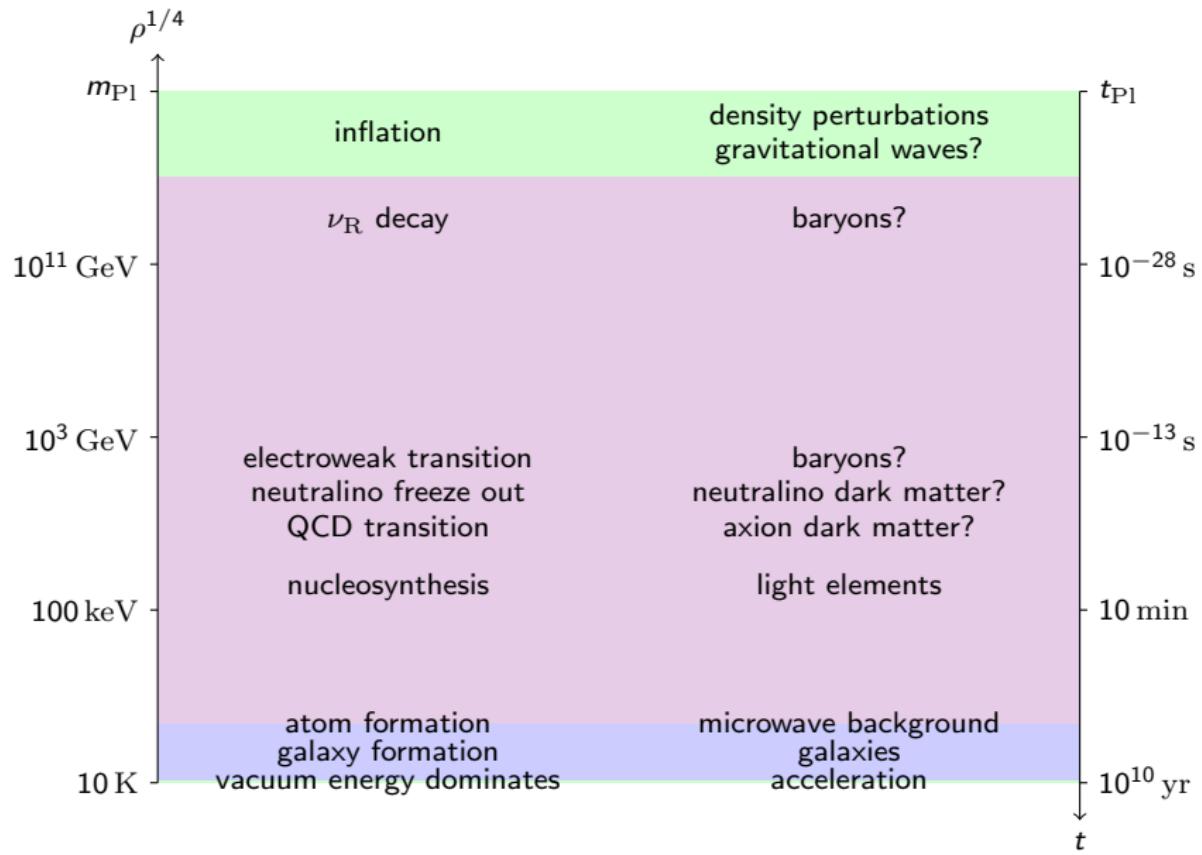
Ewan Stewart

KAIST

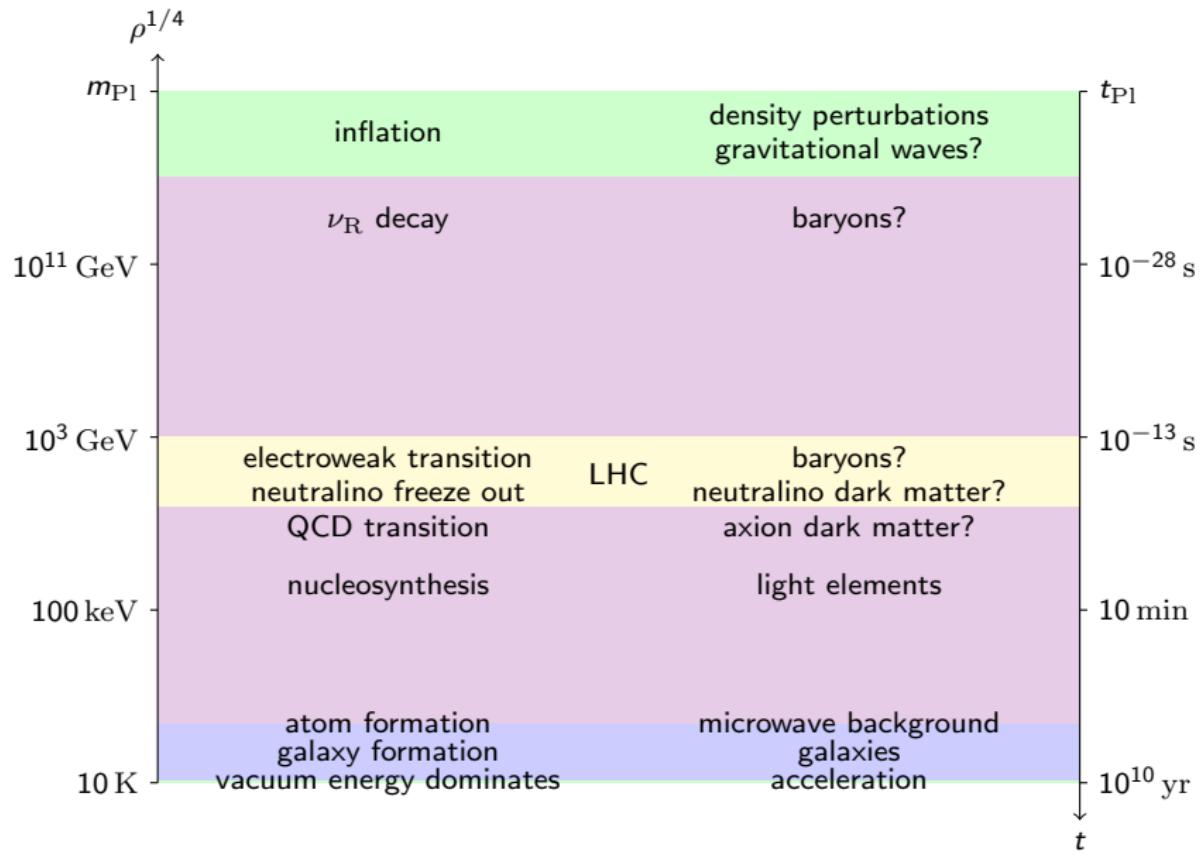
Prometeo I: LHC Physics and Cosmology
4 March 2009
Dept. of Theoretical Physics, University of Valencia

Donghui Jeong, Kenji Kadota, Wan-II Park, EDS	hep-ph/0406136
Gary N Felder, Hyunbyuk Kim, Wan-II Park, EDS	hep-ph/0703275
Richard Easterer, John T Giblin, Eugene A Lim, Wan-II Park, EDS	arXiv:0801.4197
Seongcheol Kim, Wan-II Park, EDS	arXiv:0807.3607

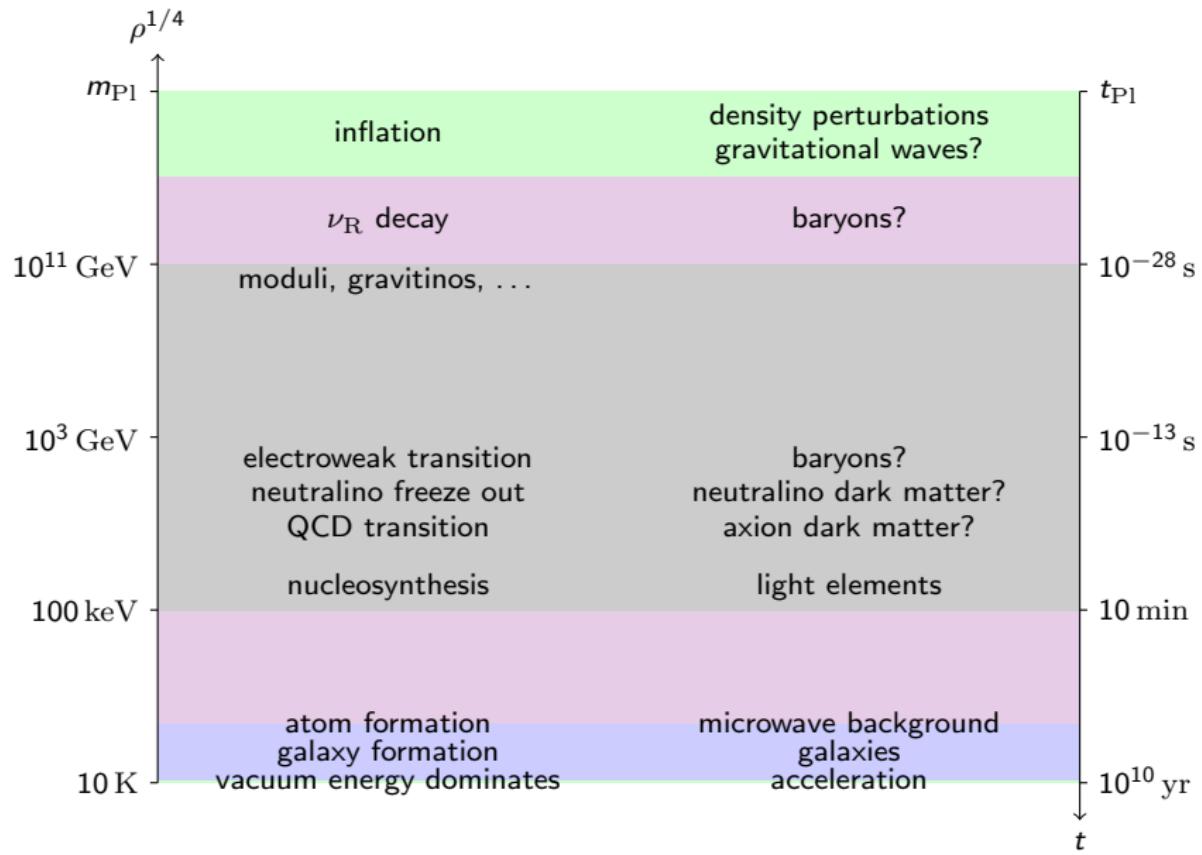
History of the observable universe



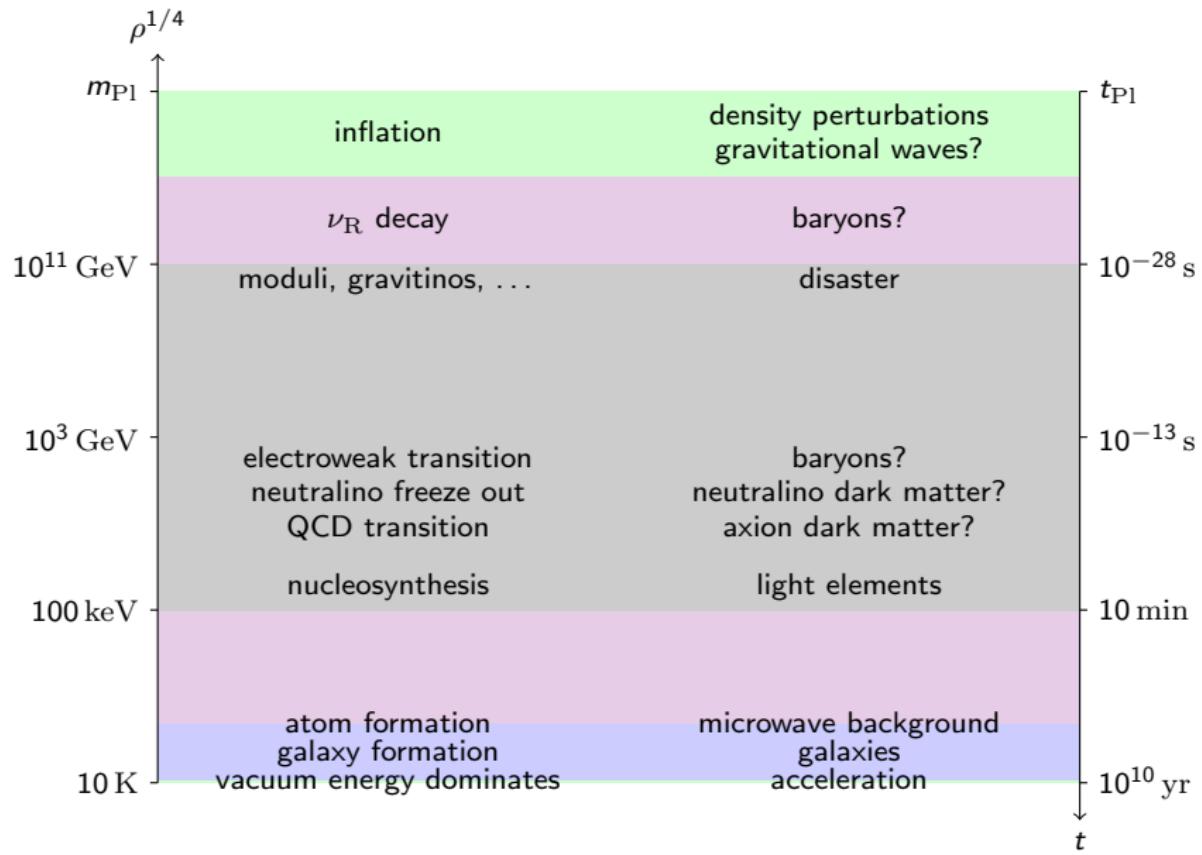
History of the observable universe



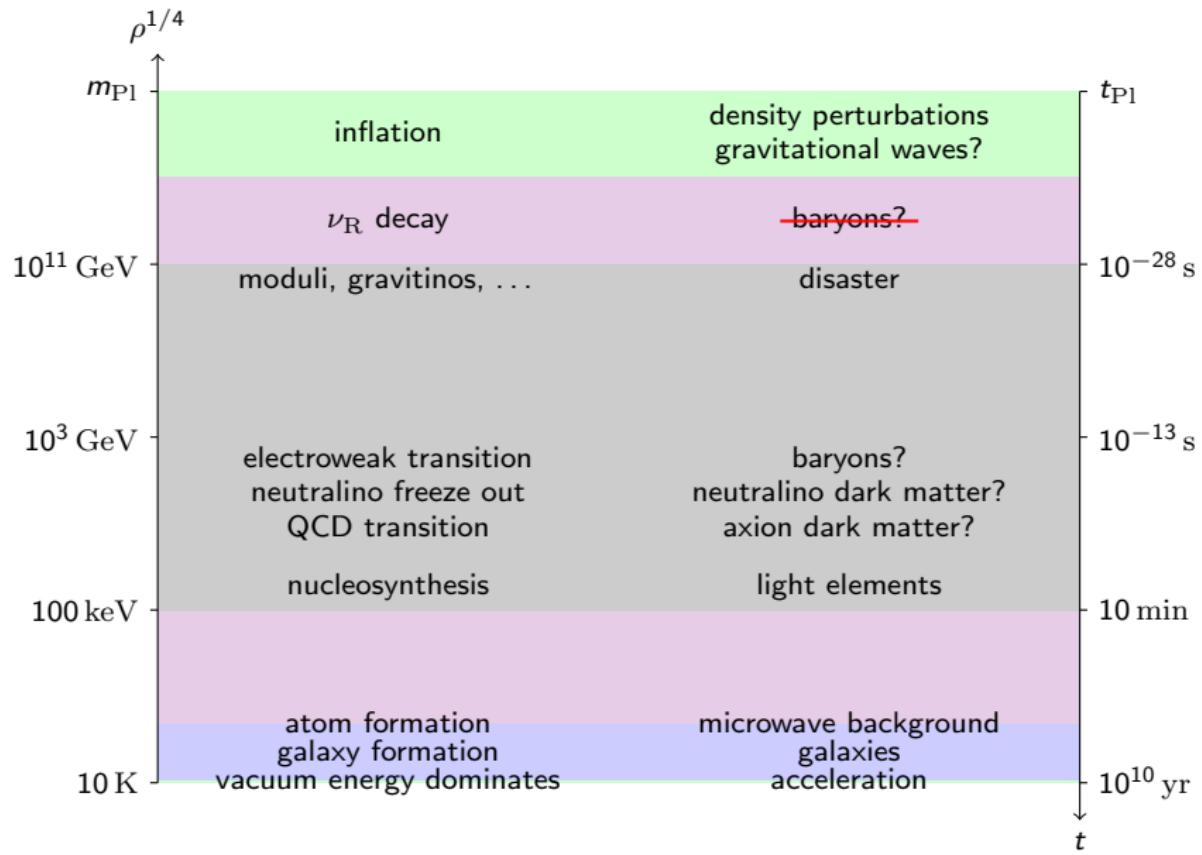
History of the observable universe



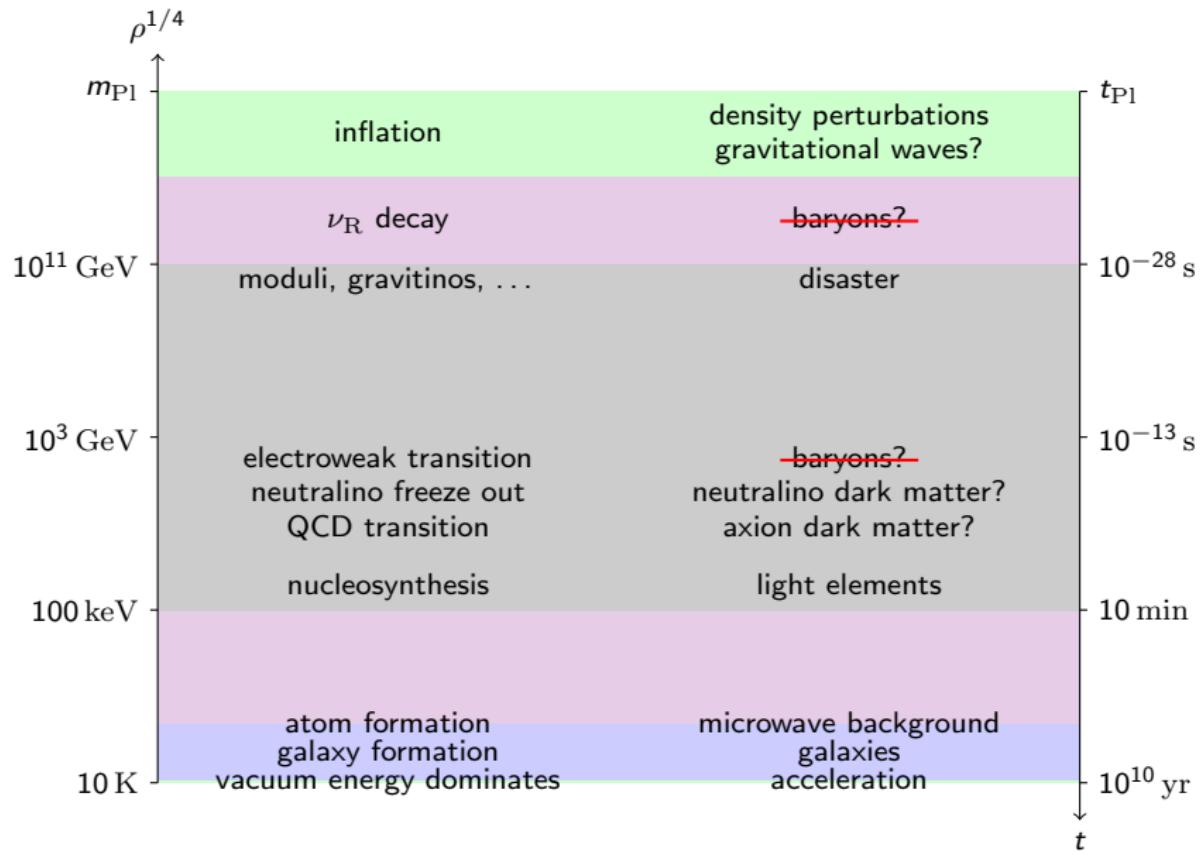
History of the observable universe



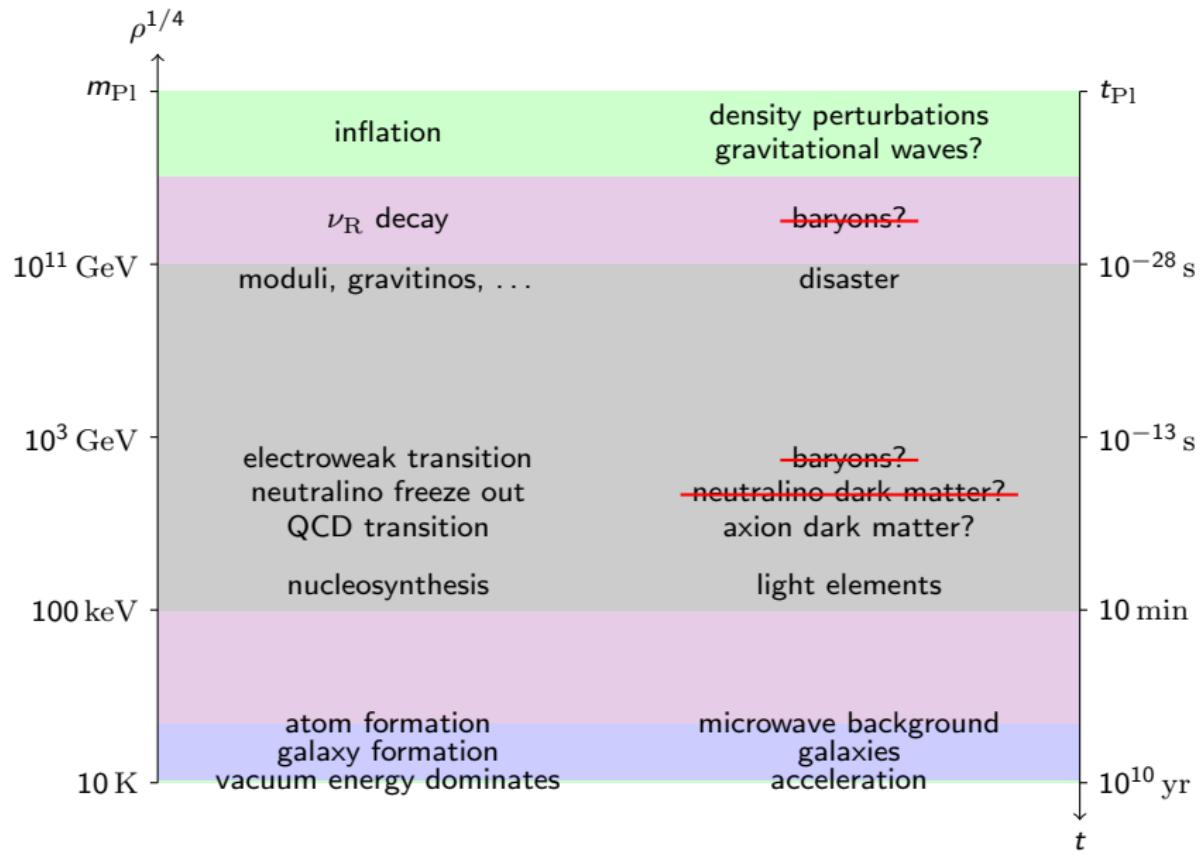
History of the observable universe



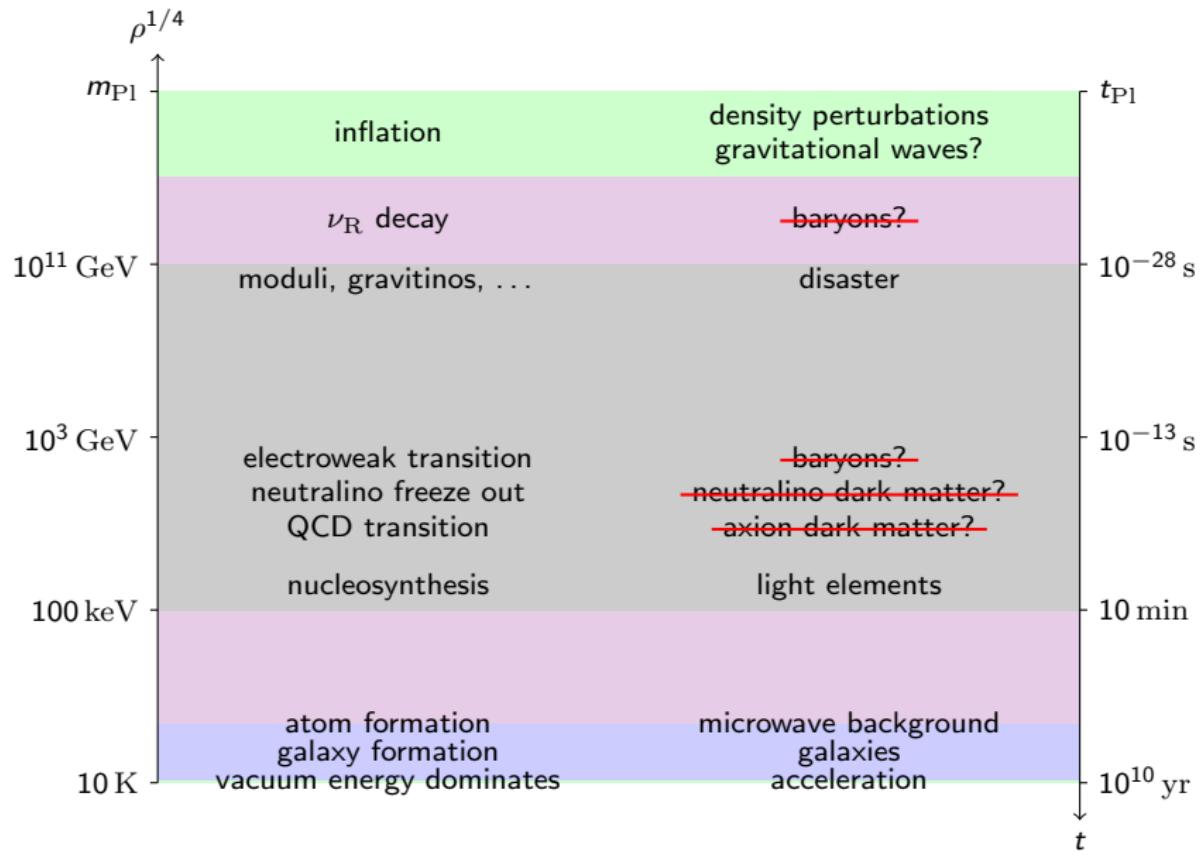
History of the observable universe



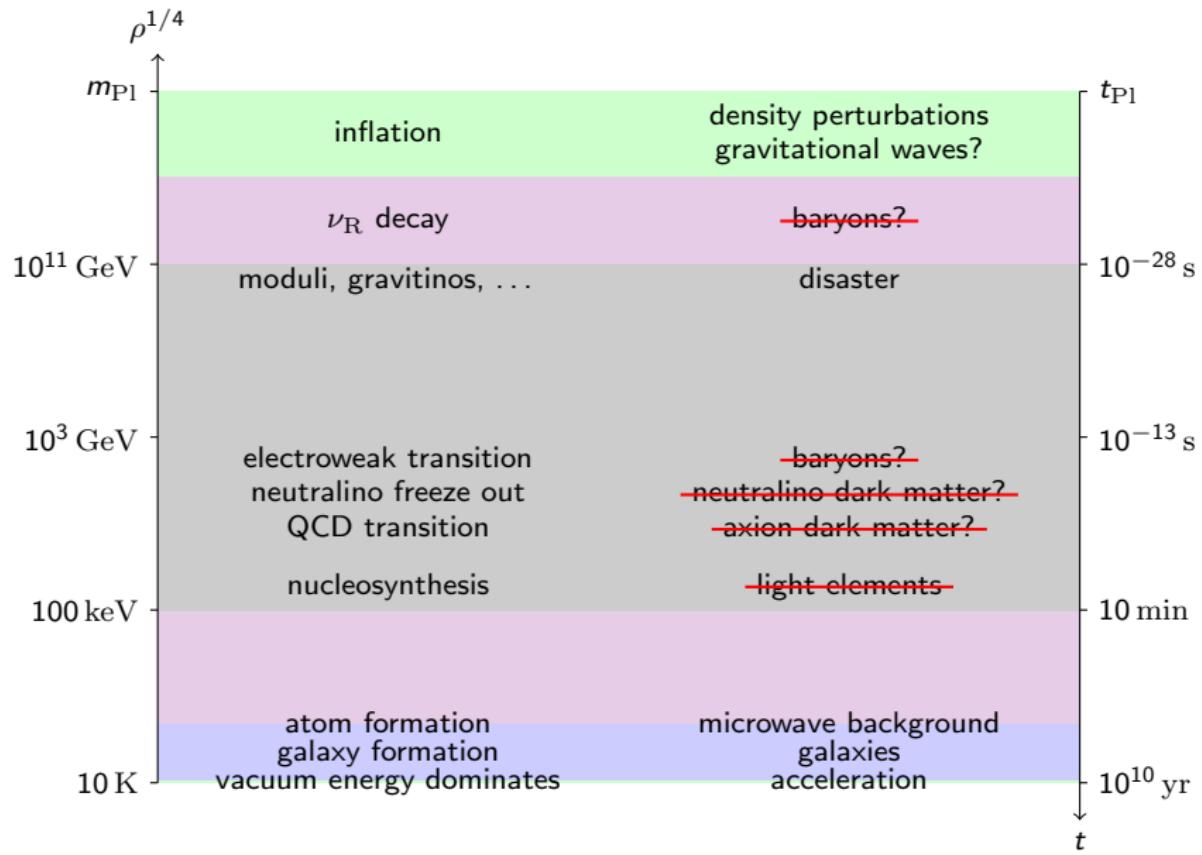
History of the observable universe



History of the observable universe



History of the observable universe



Moduli problem

Moduli (fields with Planckian expectation values) are cosmologically dangerous. For example, nucleosynthesis constrains

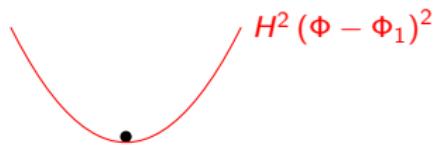
$$\frac{n_\Phi}{s} \lesssim 10^{-12}$$

Moduli problem

Moduli (fields with Planckian expectation values) are cosmologically dangerous. For example, nucleosynthesis constrains

$$\frac{n_\Phi}{s} \lesssim 10^{-12}$$

In the early universe

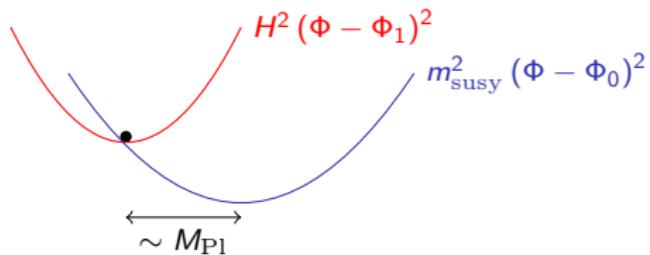


Moduli problem

Moduli (fields with Planckian expectation values) are cosmologically dangerous. For example, nucleosynthesis constrains

$$\frac{n_\Phi}{s} \lesssim 10^{-12}$$

In the early universe

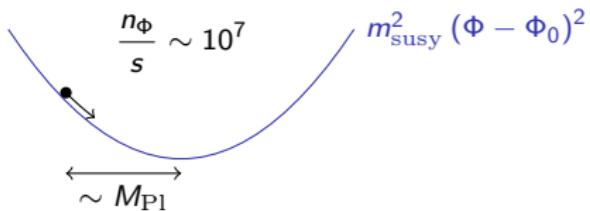


Moduli problem

Moduli (fields with Planckian expectation values) are cosmologically dangerous. For example, nucleosynthesis constrains

$$\frac{n_\Phi}{s} \lesssim 10^{-12}$$

In the early universe

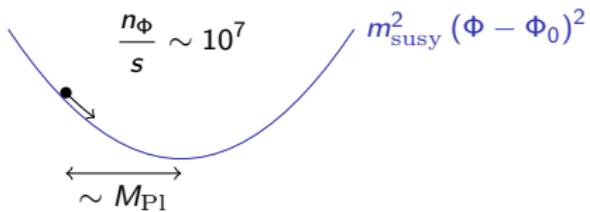


Moduli problem

Moduli (fields with Planckian expectation values) are cosmologically dangerous. For example, nucleosynthesis constrains

$$\frac{n_\Phi}{s} \lesssim 10^{-12}$$

In the early universe



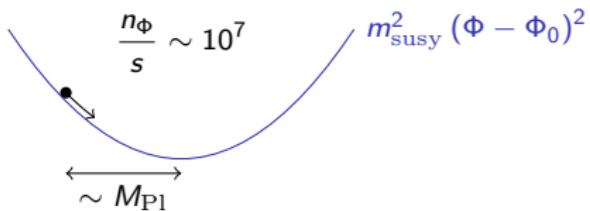
Moduli generated: $H \lesssim m_{\text{susy}}$

Moduli problem

Moduli (fields with Planckian expectation values) are cosmologically dangerous. For example, nucleosynthesis constrains

$$\frac{n_\Phi}{s} \lesssim 10^{-12}$$

In the early universe



Moduli generated: $H \lesssim m_{\text{susy}}$

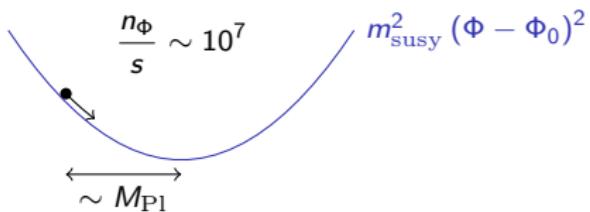
slow-roll inflation: $H \gtrsim m_{\text{inflaton}} \gtrsim m_{\text{susy}}$

Moduli problem

Moduli (fields with Planckian expectation values) are cosmologically dangerous. For example, nucleosynthesis constrains

$$\frac{n_\Phi}{s} \lesssim 10^{-12}$$

In the early universe

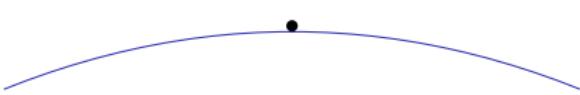


Moduli generated: $H \lesssim m_{\text{susy}}$

after

slow-roll inflation: $H \gtrsim m_{\text{inflaton}} \gtrsim m_{\text{susy}}$

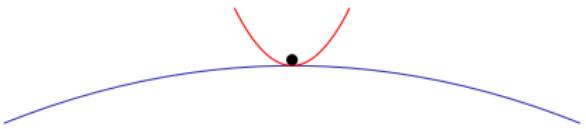
Thermal inflation



$$V = V_0$$

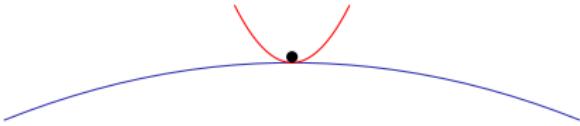
$$- m^2 |\phi|^2 + \dots$$

Thermal inflation



$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

Thermal inflation

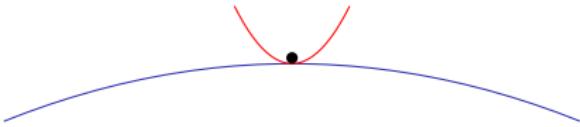


$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

Inflation for

$$V_0^{1/4} \gtrsim T \gtrsim m$$

Thermal inflation



$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

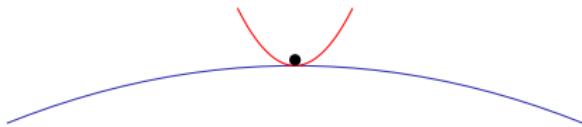
Inflation for

$$V_0^{1/4} \gtrsim T \gtrsim m$$

$T \propto e^{-N}$ so few e-folds

$$N \sim \ln \frac{V_0^{1/4}}{m}$$

Thermal inflation



If

$$V_0^{1/4} \sim 10^6 \text{ to } 10^7 \text{ GeV}$$

which for $V_0 \sim m^2 \phi_0^2$ corresponds to

$$\phi_0 \sim 10^{10} \text{ to } 10^{12} \text{ GeV}$$

then

$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

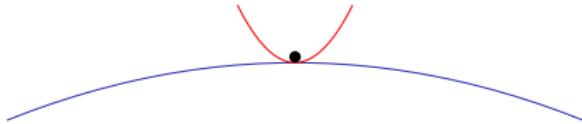
Inflation for

$$V_0^{1/4} \gtrsim T \gtrsim m$$

$T \propto e^{-N}$ so few e-folds

$$N \sim \ln \frac{V_0^{1/4}}{m}$$

Thermal inflation



If

$$V_0^{1/4} \sim 10^6 \text{ to } 10^7 \text{ GeV}$$

which for $V_0 \sim m^2 \phi_0^2$ corresponds to

$$\phi_0 \sim 10^{10} \text{ to } 10^{12} \text{ GeV}$$

then

dilution factor $\sim 10^{20}$: pre-existing moduli sufficiently diluted,

$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

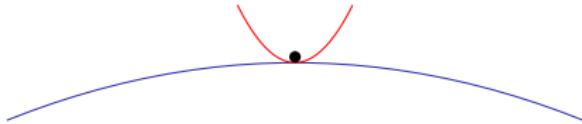
Inflation for

$$V_0^{1/4} \gtrsim T \gtrsim m$$

$$T \propto e^{-N} \text{ so few e-folds}$$

$$N \sim \ln \frac{V_0^{1/4}}{m}$$

Thermal inflation



If

$$V_0^{1/4} \sim 10^6 \text{ to } 10^7 \text{ GeV}$$

which for $V_0 \sim m^2 \phi_0^2$ corresponds to

$$\phi_0 \sim 10^{10} \text{ to } 10^{12} \text{ GeV}$$

then

dilution factor $\sim 10^{20}$: pre-existing moduli sufficiently diluted,

$H \sim 10^{-8} m$: moduli regenerated with sufficiently small abundance,

$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

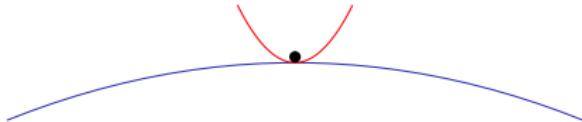
Inflation for

$$V_0^{1/4} \gtrsim T \gtrsim m$$

$$T \propto e^{-N} \text{ so few e-folds}$$

$$N \sim \ln \frac{V_0^{1/4}}{m}$$

Thermal inflation



If

$$V_0^{1/4} \sim 10^6 \text{ to } 10^7 \text{ GeV}$$

which for $V_0 \sim m^2 \phi_0^2$ corresponds to

$$\phi_0 \sim 10^{10} \text{ to } 10^{12} \text{ GeV}$$

then

dilution factor $\sim 10^{20}$: pre-existing moduli sufficiently diluted,

$H \sim 10^{-8}m$: moduli regenerated with sufficiently small abundance,

$N \sim 10$: primordial perturbations from slow-roll inflation preserved on large scales,

$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

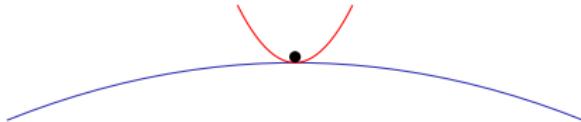
Inflation for

$$V_0^{1/4} \gtrsim T \gtrsim m$$

$T \propto e^{-N}$ so few e-folds

$$N \sim \ln \frac{V_0^{1/4}}{m}$$

Thermal inflation



If

$$V_0^{1/4} \sim 10^6 \text{ to } 10^7 \text{ GeV}$$

which for $V_0 \sim m^2 \phi_0^2$ corresponds to

$$\phi_0 \sim 10^{10} \text{ to } 10^{12} \text{ GeV}$$

then

dilution factor $\sim 10^{20}$: pre-existing moduli sufficiently diluted,

$H \sim 10^{-8}m$: moduli regenerated with sufficiently small abundance,

$N \sim 10$: primordial perturbations from slow-roll inflation preserved on large scales,

$H \sim 1 \text{ to } 10 \text{ keV}$: primordial gravitational waves wiped out on solar system scales.

$$V = V_0 + g^2 T^2 |\phi|^2 - m^2 |\phi|^2 + \dots$$

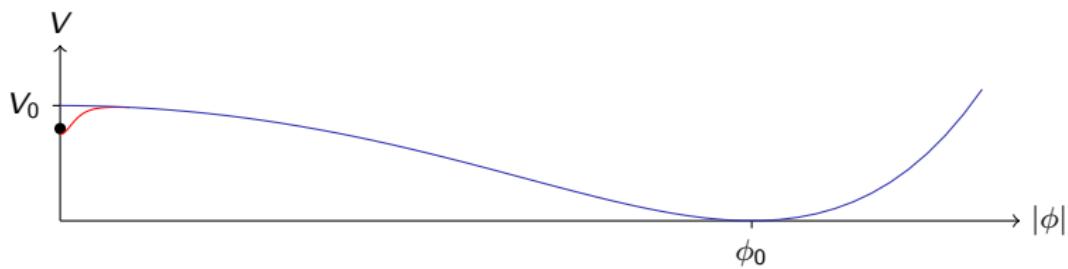
Inflation for

$$V_0^{1/4} \gtrsim T \gtrsim m$$

$$T \propto e^{-N} \text{ so few e-folds}$$

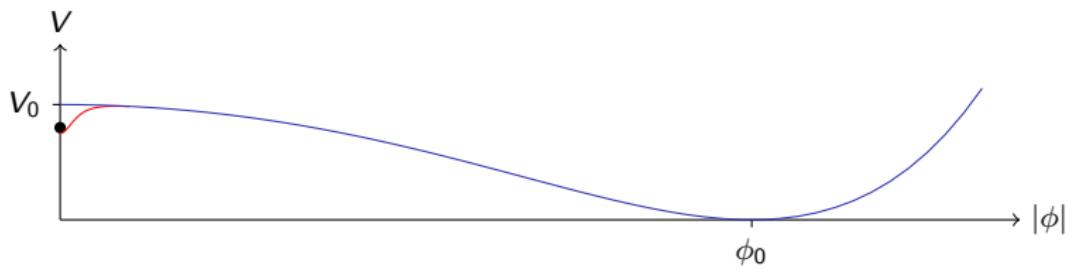
$$N \sim \ln \frac{V_0^{1/4}}{m}$$

First order phase transition



First order phase transition since $\phi_0 \gg T_c \sim m$.

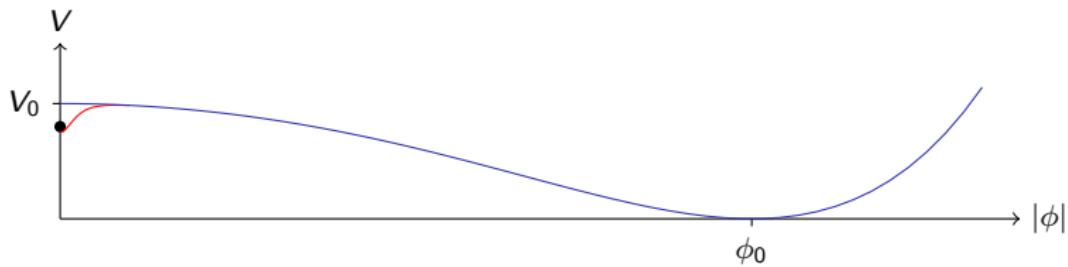
First order phase transition



First order phase transition since $\phi_0 \gg T_c \sim m$. Typical bubble size

$$\frac{\Gamma}{\dot{\Gamma}} \sim (10^{-3} \text{ to } 10^{-5}) \frac{1}{H} \sim (10^5 \text{ to } 10^3) \frac{1}{m}$$

First order phase transition



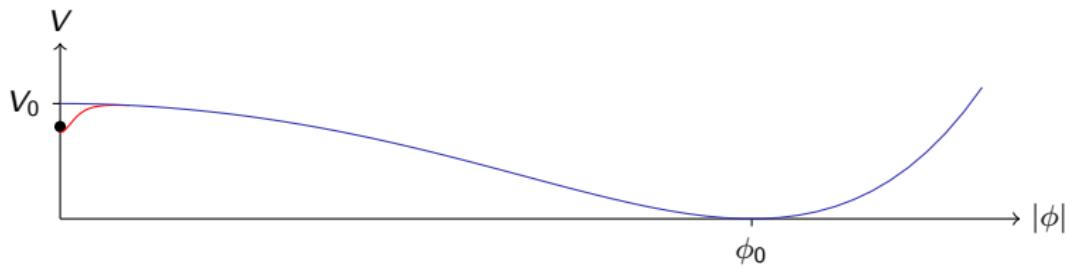
First order phase transition since $\phi_0 \gg T_c \sim m$. Typical bubble size

$$\frac{\Gamma}{\dot{\Gamma}} \sim (10^{-3} \text{ to } 10^{-5}) \frac{1}{H} \sim (10^5 \text{ to } 10^3) \frac{1}{m}$$

Gravitational waves generated with frequency

$$f \sim 10 \text{ Hz} \left(\frac{\dot{\Gamma}/H\Gamma}{10^4} \right) \left(\frac{V_0^{1/4}}{10^{6.5} \text{ GeV}} \right)^{2/3} \left(\frac{T_d}{10 \text{ GeV}} \right)^{1/3} \left(\frac{V_0 a_c^3 / \rho_d a_d^3}{10} \right)^{1/3}$$

First order phase transition



First order phase transition since $\phi_0 \gg T_c \sim m$. Typical bubble size

$$\frac{\Gamma}{\dot{\Gamma}} \sim (10^{-3} \text{ to } 10^{-5}) \frac{1}{H} \sim (10^5 \text{ to } 10^3) \frac{1}{m}$$

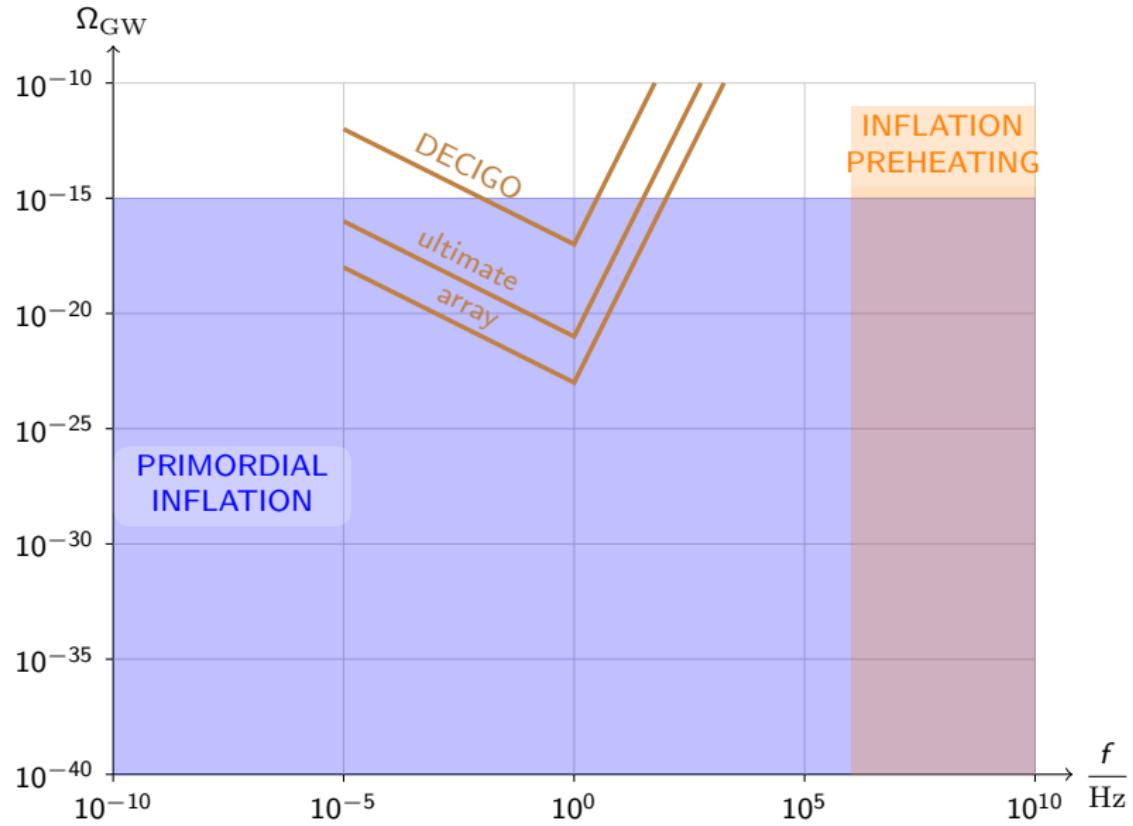
Gravitational waves generated with frequency

$$f \sim 10 \text{ Hz} \left(\frac{\dot{\Gamma}/H\Gamma}{10^4} \right) \left(\frac{V_0^{1/4}}{10^{6.5} \text{ GeV}} \right)^{2/3} \left(\frac{T_d}{10 \text{ GeV}} \right)^{1/3} \left(\frac{V_0 a_c^3 / \rho_d a_d^3}{10} \right)^{1/3}$$

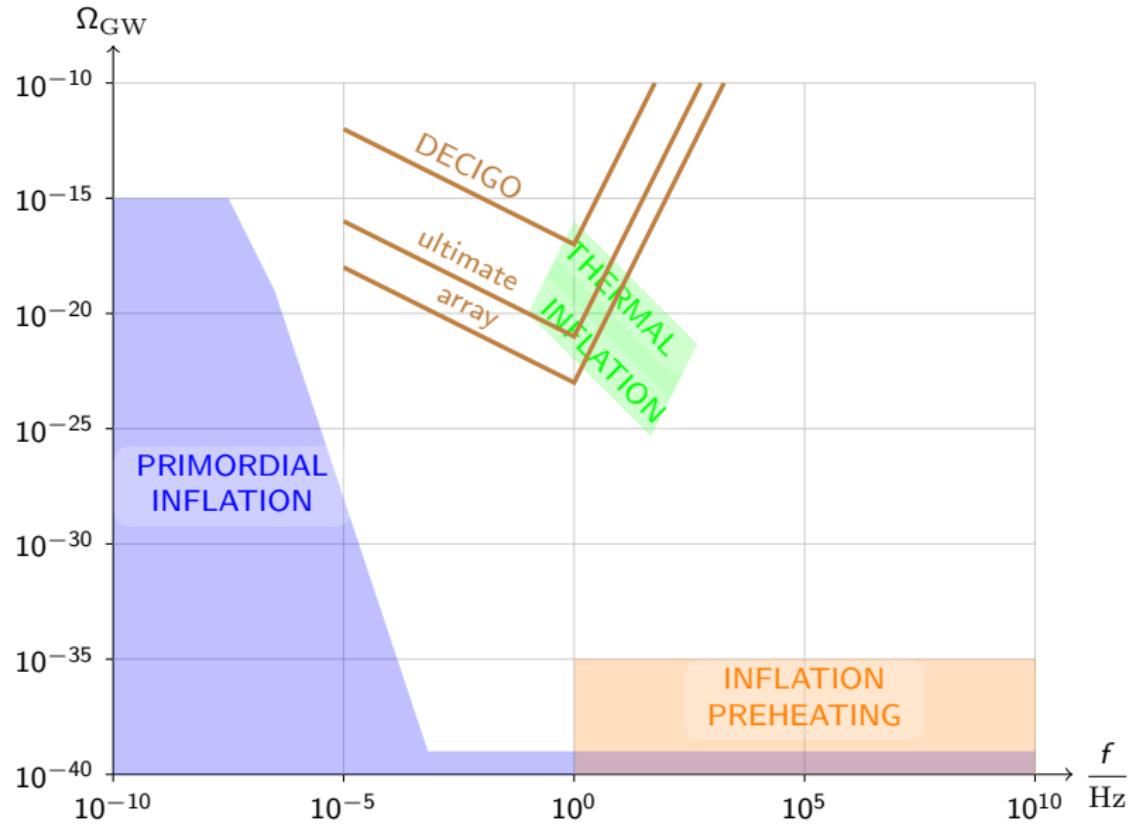
and density

$$\Omega_{\text{GW}} \sim 10^{-20} \left(\frac{10^4}{\dot{\Gamma}/H\Gamma} \right)^2 \left(\frac{10^{6.5} \text{ GeV}}{V_0^{1/4}} \right)^{4/3} \left(\frac{T_d}{10 \text{ GeV}} \right)^{4/3} \left(\frac{V_0 a_c^3 / \rho_d a_d^3}{10} \right)^{4/3}$$

Gravitational waves



Gravitational waves



MSCM superpotential

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d$$

MSCM superpotential

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d + \frac{1}{2} \lambda_\nu (L H_u)^2$$

MSCM superpotential

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \cancel{\mu H_u H_d} + \frac{1}{2} \lambda_\nu (L H_u)^2 + \lambda_\mu \phi^2 H_u H_d$$

$$\mu = \lambda_\mu \phi_0^2$$

MSCM superpotential

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \frac{1}{2} \lambda_\nu (L H_u)^2 + \lambda_\mu \phi^2 H_u H_d$$

$$\mu = \lambda_\mu \phi_0^2$$

MSCM superpotential

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \frac{1}{2} \lambda_\nu (L H_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

$$\mu = \lambda_\mu \phi_0^2$$

$$m_\phi^2 < 0$$

MSCM superpotential

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \frac{1}{2} \lambda_\nu (L H_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

$$\mu = \lambda_\mu \phi_0^2$$

$$m_\phi^2 < 0$$

Key assumption

$$m_{LH_u}^2 = \frac{1}{2} (m_L^2 + m_{H_u}^2) < 0$$

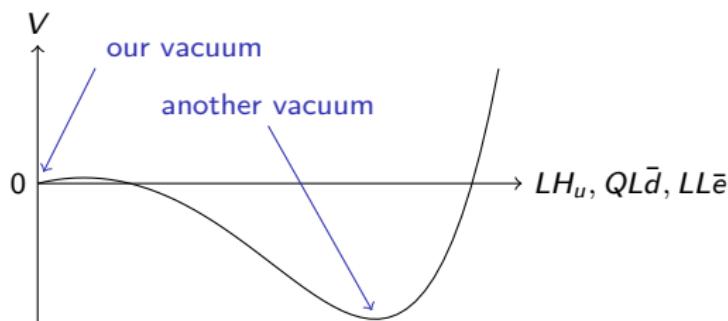
Key assumption

$$m_{LH_u}^2 = \frac{1}{2} (m_L^2 + m_{H_u}^2) < 0$$

Implies a dangerous non-MSSM vacuum with $LH_u \sim (10^9 \text{GeV})^2$ and

$$\lambda_d Q L \bar{d} + \lambda_e L L \bar{e} = -\mu LH_u$$

eliminating the μ -term contribution to LH_u 's mass squared.



Reduction

For simplicity, reduce to a single generation

$$L = \begin{pmatrix} & & \end{pmatrix}, \quad H_u = \begin{pmatrix} & & \end{pmatrix}, \quad H_d = \begin{pmatrix} & & \end{pmatrix}, \quad \bar{e} = (\quad \quad \quad)$$

$$\bar{u} = (\quad \quad \quad) , \quad Q = \begin{pmatrix} & & \end{pmatrix}, \quad \bar{d} = (\quad \quad \quad)$$

$$\phi = \quad , \quad \chi = \quad , \quad \bar{\chi} = \quad$$

Reduction

For simplicity, reduce to a single generation

$$L = \begin{pmatrix} & \\ & I \end{pmatrix} , \quad H_u = \begin{pmatrix} h_u \\ 0 \end{pmatrix} , \quad H_d = \begin{pmatrix} & \\ & \end{pmatrix} , \quad \bar{e} = (\quad)$$

$$\bar{u} = (\quad) , \quad Q = \begin{pmatrix} & \\ & \end{pmatrix} , \quad \bar{d} = (\quad)$$

$$\phi = \phi , \quad \chi = , \quad \bar{\chi} =$$

Reduction

For simplicity, reduce to a single generation

$$L = \begin{pmatrix} & \\ & I & \end{pmatrix} , \quad H_u = \begin{pmatrix} h_u \\ 0 \end{pmatrix} , \quad H_d = \begin{pmatrix} & 0 \end{pmatrix} , \quad \bar{e} = (\quad)$$

$$\bar{u} = (\ 0 \ 0 \ 0 \) , \quad Q = \begin{pmatrix} & & \\ & 0 & 0 \end{pmatrix} , \quad \bar{d} = (\quad)$$

$$\phi = \phi , \quad \chi = 0 , \quad \bar{\chi} = 0$$

Reduction

For simplicity, reduce to a single generation

$$L = \begin{pmatrix} & \\ & I & \end{pmatrix} , \quad H_u = \begin{pmatrix} h_u \\ 0 \end{pmatrix} , \quad H_d = \begin{pmatrix} 0 \\ h_d \end{pmatrix} , \quad \bar{e} = (e/\sqrt{2})$$

$$\bar{u} = (0 \ 0 \ 0) , \quad Q = \begin{pmatrix} & & \\ & 0 & 0 \\ & 0 & 0 \end{pmatrix} , \quad \bar{d} = (d/\sqrt{2} \ 0 \ 0)$$

$$\phi = \phi , \quad \chi = 0 , \quad \bar{\chi} = 0$$

Reduction

For simplicity, reduce to a single generation

$$L = \begin{pmatrix} e/\sqrt{2} \\ I \end{pmatrix} , \quad H_u = \begin{pmatrix} h_u \\ 0 \end{pmatrix} , \quad H_d = \begin{pmatrix} 0 \\ h_d \end{pmatrix} , \quad \bar{e} = (e/\sqrt{2})$$

$$\bar{u} = (0 \ 0 \ 0) , \quad Q = \begin{pmatrix} d/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad \bar{d} = (d/\sqrt{2} \ 0 \ 0)$$

$$\phi = \phi , \quad \chi = 0 , \quad \bar{\chi} = 0$$

Reduction

For simplicity, reduce to a single generation

$$L = \begin{pmatrix} e/\sqrt{2} \\ I \end{pmatrix} , \quad H_u = \begin{pmatrix} h_u \\ 0 \end{pmatrix} , \quad H_d = \begin{pmatrix} 0 \\ h_d \end{pmatrix} , \quad \bar{e} = (e/\sqrt{2})$$

$$\bar{u} = (0 \ 0 \ 0) , \quad Q = \begin{pmatrix} d/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad \bar{d} = (d/\sqrt{2} \ 0 \ 0)$$

$$\phi = \phi , \quad \chi = 0 , \quad \bar{\chi} = 0$$

The superpotential reduces to

$$W = \frac{1}{2} \lambda_d h_d d^2 + \frac{1}{2} \lambda_e h_d e^2 + \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} \lambda_\nu (I h_u)^2$$

with the remaining D -term constraint

$$D = |h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 = 0$$

Potential

$$\begin{aligned} V = & V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2 \\ & + \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2 \\ & + \left[\frac{1}{2} A_\nu \lambda_\nu l^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right] \\ & + \left| \lambda_\nu l h_u^2 \right|^2 + \left| \lambda_\nu l^2 h_u + \lambda_\mu \phi^2 h_d \right|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2 \\ & + |\lambda_d h_d d|^2 + |\lambda_e h_d e|^2 + |2\lambda_\mu \phi h_u h_d|^2 \\ & + \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2 \end{aligned}$$

Potential

drives thermal inflation

$$\begin{aligned} V = & V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2 \\ & + \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2 \\ & + \left[\frac{1}{2} A_\nu \lambda_\nu l^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right] \\ & + \left| \lambda_\nu l h_u^2 \right|^2 + \left| \lambda_\nu l^2 h_u + \lambda_\mu \phi^2 h_d \right|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2 \\ & + \left| \lambda_d h_d d \right|^2 + \left| \lambda_e h_d e \right|^2 + \left| 2 \lambda_\mu \phi h_u h_d \right|^2 \\ & + \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2 \end{aligned}$$

Potential

drives thermal inflation

$I h_u$ rolls away

$$\begin{aligned} V = & V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2 \\ & + \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2 \\ & + \left[\frac{1}{2} A_\nu \lambda_\nu I^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right] \\ & + |\lambda_\nu I h_u^2|^2 + \left| \lambda_\nu I^2 h_u + \lambda_\mu \phi^2 h_d \right|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2 \\ & + |\lambda_d h_d d|^2 + |\lambda_e h_d e|^2 + |2 \lambda_\mu \phi h_u h_d|^2 \\ & + \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2 \end{aligned}$$

Potential

drives thermal inflation

Ih_u rolls away

$$V = V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2$$

$$+ \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2$$

Ih_u stabilized
with
fixed phase

$$+ \left[\frac{1}{2} A_\nu \lambda_\nu l^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right]$$

$$+ |\lambda_\nu I h_u^2|^2 + |\lambda_\nu l^2 h_u + \lambda_\mu \phi^2 h_d|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2$$

$$+ |\lambda_d h_d d|^2 + |\lambda_e h_d e|^2 + |2 \lambda_\mu \phi h_u h_d|^2$$

$$+ \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2$$

Potential

drives thermal inflation

lh_u rolls away

ϕ rolls away

$$V = V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2$$

$$+ \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2$$

lh_u stabilized
with
fixed phase

$$+ \left[\frac{1}{2} A_\nu \lambda_\nu l^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right]$$

$$+ |\lambda_\nu lh_u^2|^2 + |\lambda_\nu l^2 h_u + \lambda_\mu \phi^2 h_d|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2$$

$$+ |\lambda_d h_d d|^2 + |\lambda_e h_d e|^2 + |2\lambda_\mu \phi h_u h_d|^2$$

$$+ \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2$$

Potential

$$\begin{aligned}
 V &= V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2 \\
 &\quad + \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2 \\
 &\quad + \left[\frac{1}{2} A_\nu \lambda_\nu l^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right] \\
 &\quad + |\lambda_\nu l h_u^2|^2 + \left| \lambda_\nu l^2 h_u + \lambda_\mu \phi^2 h_d \right|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2 \\
 &\quad + |\lambda_d h_d d|^2 + |\lambda_e h_d e|^2 + |2\lambda_\mu \phi h_u h_d|^2 \\
 &\quad + \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2
 \end{aligned}$$

drives thermal inflation
\$lh_u\$ rolls away
\$\phi\$ rolls away

\$h_d\$ forced out

\$lh_u\$ stabilized with fixed phase

Potential

drives thermal inflation h_u rolls away ϕ rolls away

$$V = V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2$$
$$+ \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2$$
$$+ \left[\frac{1}{2} A_\nu \lambda_\nu I^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right]$$

h_d forced out

h_u stabilized with fixed phase

$$+ |\lambda_\nu I h_u^2|^2 + \left| \lambda_\nu I^2 h_u + \lambda_\mu \phi^2 h_d \right|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2$$
$$+ |\lambda_d h_d d|^2 + |\lambda_e h_d e|^2 + |2 \lambda_\mu \phi h_u h_d|^2$$
$$+ \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2$$

d and e held at origin

Potential

drives thermal inflation h_u rolls away ϕ rolls away

$$\begin{aligned}
 V = & V_0 + m_L^2 |I|^2 - m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 - m_\phi^2 |\phi|^2 \\
 & + \frac{1}{2} (m_Q^2 + m_d^2) |d|^2 + \frac{1}{2} (m_L^2 + m_e^2) |e|^2 \\
 & + \left[\frac{1}{2} A_\nu \lambda_\nu I^2 h_u^2 + A_\mu \lambda_\mu \phi^2 h_u h_d + \frac{1}{2} A_d \lambda_d h_d d^2 + \frac{1}{2} A_e \lambda_e h_d e^2 + \text{c.c.} \right] \\
 & + |\lambda_\nu I h_u^2|^2 + \left| \lambda_\nu I^2 h_u + \lambda_\mu \phi^2 h_d \right|^2 + \left| \lambda_\mu \phi^2 h_u + \frac{1}{2} \lambda_d d^2 + \frac{1}{2} \lambda_e e^2 \right|^2 \\
 & + |\lambda_d h_d d|^2 + |\lambda_e h_d e|^2 + |2\lambda_\mu \phi h_u h_d|^2 \\
 & + \frac{1}{2} g^2 \left(|h_u|^2 - |h_d|^2 - |I|^2 + \frac{1}{2} |d|^2 + \frac{1}{2} |e|^2 \right)^2
 \end{aligned}$$

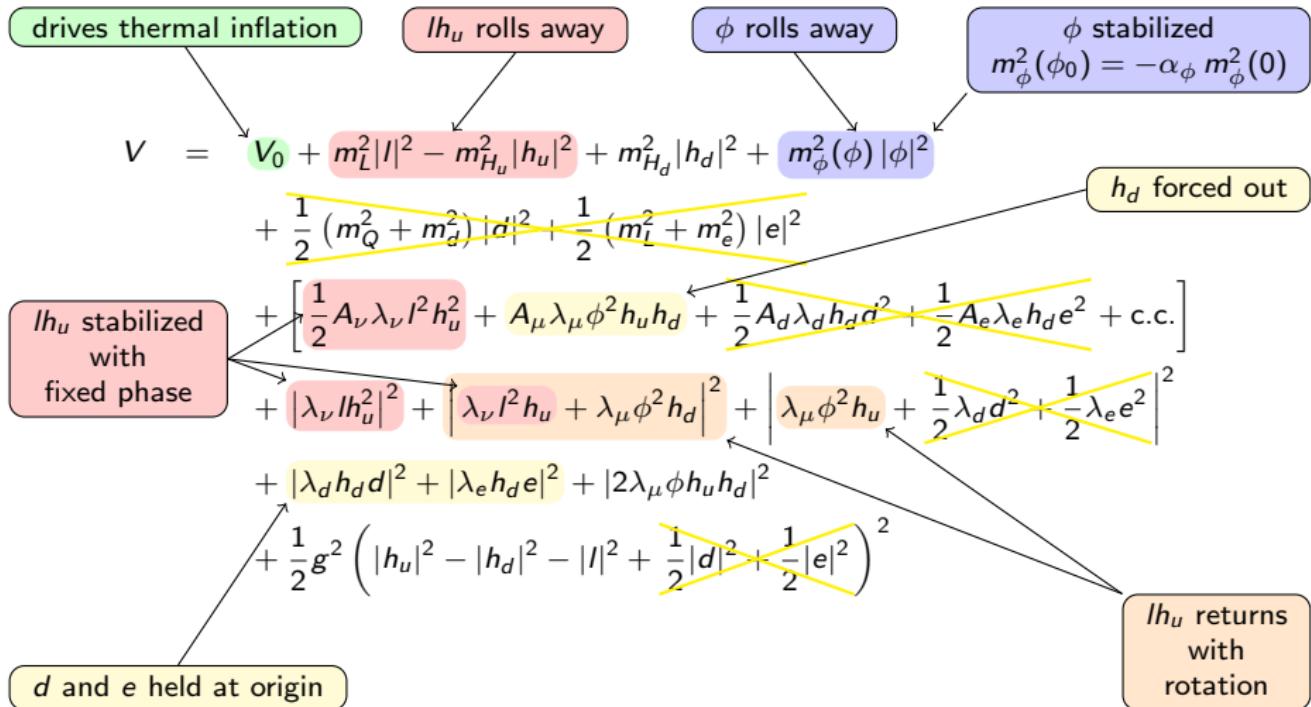
h_d forced out

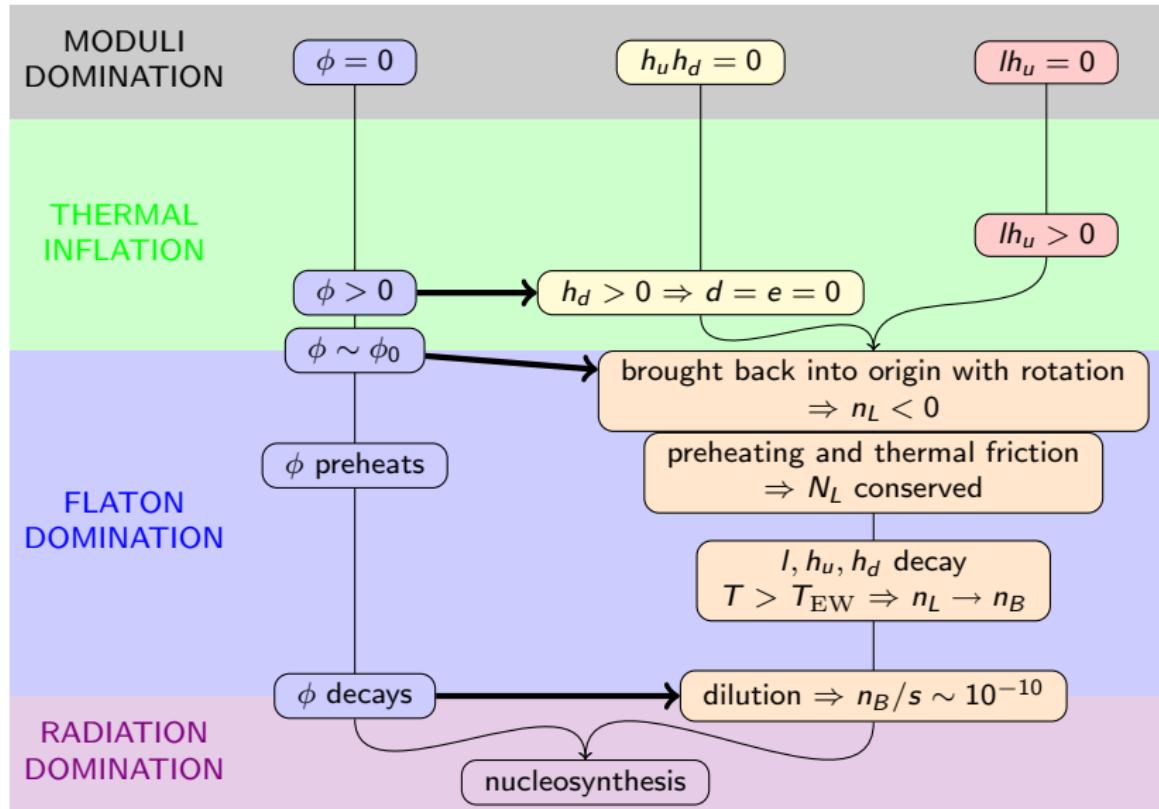
h_u stabilized with fixed phase

d and e held at origin

h_u returns with rotation

Potential





Simulation

Lattice 128^3 , box size = $200m^{-1}$, Fourier modes $0.033m \leq k \leq 3.5m$.

CP phase

$$\arg(-B^* A_\nu) = \begin{cases} \pi - \frac{\pi}{20} & CP+ \\ \pi & CP0 \\ \pi + \frac{\pi}{20} & CP- \end{cases}$$

Initial conditions

$$\phi = 4m + \delta\phi$$

$$l = l_0 + \delta l$$

$$h_d = \delta h_d$$

Constraints

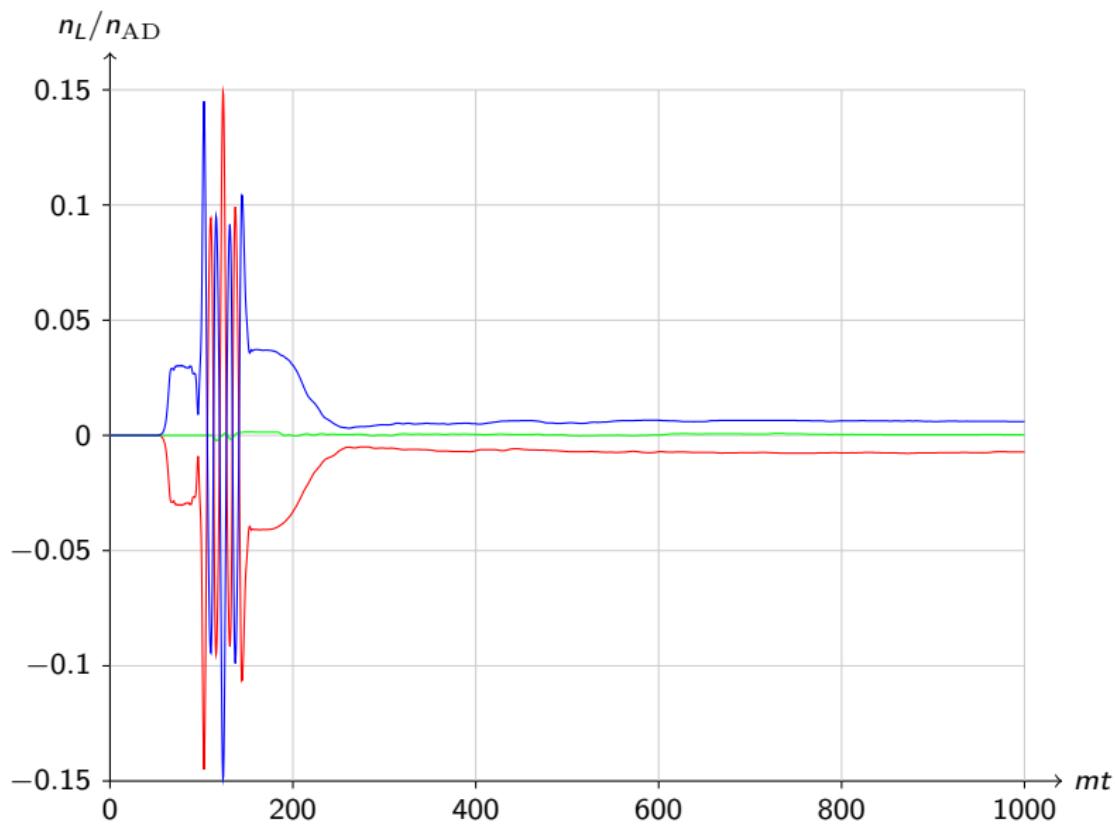
$$D = \epsilon^2 \quad \text{with } \epsilon = 4.8 \times 10^{-3} l_0$$

$$j_0 = 0$$

Algorithm Adaptive constrained gauge invariant leapfrog type algorithm.

Exactly conserves the constraints and charges, and has good energy conservation.

Lepton number



Baryon asymmetry

$$\frac{n_B}{s} \sim \frac{n_L}{n_{\text{AD}}} \frac{n_{\text{AD}}}{n_\phi} \frac{T_{\text{d}}}{m_\phi(\phi_0)}$$

Baryon asymmetry

$$\frac{n_B}{s} \sim \frac{n_L}{n_{\text{AD}}} \frac{n_{\text{AD}}}{n_\phi} \frac{T_d}{m_\phi(\phi_0)}$$

Using $n_\phi \sim m_\phi(\phi_0) \phi_0^2$ and $m_\phi^2(\phi_0) \sim \alpha_\phi m_\phi^2(0)$, and $n_{\text{AD}} \sim m_{LH_u} l_0^2$ and

$$l_0 \sim 100 \text{ GeV} \sqrt{\frac{m_{LH_u}}{m_\nu}}$$

gives

$$\frac{n_B}{s} \sim 10^{-10} \left(\frac{n_L/n_{\text{AD}}}{10^{-2}} \right) \left(\frac{10^{12} \text{ GeV}}{\phi_0} \right)^2 \left(\frac{T_d}{1 \text{ GeV}} \right) \left(\frac{10^{-1}}{\alpha_\phi} \right) \left(\frac{10^{-2} \text{ eV}}{m_\nu} \right) \left(\frac{m_{LH_u}}{m_\phi(0)} \right)^2$$

Baryon asymmetry

$$\frac{n_B}{s} \sim \frac{n_L}{n_{\text{AD}}} \frac{n_{\text{AD}}}{n_\phi} \frac{T_d}{m_\phi(\phi_0)}$$

Using $n_\phi \sim m_\phi(\phi_0) \phi_0^2$ and $m_\phi^2(\phi_0) \sim \alpha_\phi m_\phi^2(0)$, and $n_{\text{AD}} \sim m_{LH_u} l_0^2$ and

$$l_0 \sim 100 \text{ GeV} \sqrt{\frac{m_{LH_u}}{m_\nu}}$$

gives

$$\frac{n_B}{s} \sim 10^{-10} \left(\frac{n_L/n_{\text{AD}}}{10^{-2}} \right) \left(\frac{10^{12} \text{ GeV}}{\phi_0} \right)^2 \left(\frac{T_d}{1 \text{ GeV}} \right) \left(\frac{10^{-1}}{\alpha_\phi} \right) \left(\frac{10^{-2} \text{ eV}}{m_\nu} \right) \left(\frac{m_{LH_u}}{m_\phi(0)} \right)^2$$

and using

$$T_d \sim 1 \text{ GeV} \left(\frac{10^{12} \text{ GeV}}{\phi_0} \right) \left(\frac{|\mu|}{10^3 \text{ GeV}} \right)^2$$

gives

$$\frac{n_B}{s} \sim 10^{-10} \left(\frac{n_L/n_{\text{AD}}}{10^{-2}} \right) \left(\frac{10^{12} \text{ GeV}}{\phi_0} \right)^3 \left(\frac{|\mu|}{10^3 \text{ GeV}} \right)^2 \left(\frac{10^{-1}}{\alpha_\phi} \right) \left(\frac{10^{-2} \text{ eV}}{m_\nu} \right) \left(\frac{m_{LH_u}}{m_\phi(0)} \right)^2$$

Dark matter candidates

Peccei-Quinn symmetry

$$W = \lambda_u QH_u\bar{u} + \lambda_d QH_d\bar{d} + \lambda_e LH_d\bar{e} + \frac{1}{2}\lambda_\nu (LH_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

Dark matter candidates

Peccei-Quinn symmetry

$$W = \lambda_u QH_u\bar{u} + \lambda_d QH_d\bar{d} + \lambda_e LH_d\bar{e} + \frac{1}{2}\lambda_\nu (LH_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

DFSZ axion

Dark matter candidates

Peccei-Quinn symmetry

$$W = \lambda_u QH_u\bar{u} + \lambda_d QH_d\bar{d} + \lambda_e LH_d\bar{e} + \frac{1}{2}\lambda_\nu (LH_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

DFSZ axion

KSVZ axion



Dark matter candidates

Peccei-Quinn symmetry

$$W = \lambda_u QH_u\bar{u} + \lambda_d QH_d\bar{d} + \lambda_e LH_d\bar{e} + \frac{1}{2}\lambda_\nu (LH_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

DFSZ axion

KSVZ axion

Axion

$$\begin{aligned} m_a &\sim \frac{\Lambda_{\text{QCD}}^2}{f_a} \quad \text{where } f_a = \frac{\sqrt{2} \phi_0}{N} \\ &\simeq 6.2 \times 10^{-5} \text{ eV} \left(\frac{10^{11} \text{ GeV}}{f_a} \right) \end{aligned}$$

Dark matter candidates

Peccei-Quinn symmetry

$$W = \lambda_u QH_u\bar{u} + \lambda_d QH_d\bar{d} + \lambda_e LH_d\bar{e} + \frac{1}{2}\lambda_\nu (LH_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

DFSZ axion

KSVZ axion

Axion

$$\begin{aligned} m_a &\sim \frac{\Lambda_{\text{QCD}}^2}{f_a} \quad \text{where } f_a = \frac{\sqrt{2} \phi_0}{N} \\ &\simeq 6.2 \times 10^{-5} \text{ eV} \left(\frac{10^{11} \text{ GeV}}{f_a} \right) \end{aligned}$$

Axino

$$\begin{aligned} m_{\tilde{a}} &= \frac{1}{16\pi^2} \sum_x \lambda_x^2 A_x \\ &\sim \text{1 to 10 GeV} \end{aligned}$$

Dark matter abundance

Axion

Axino

Dark matter abundance

Axion Misalignment

$$\Omega_a \sim 0.1 \left(\frac{\sqrt{6}}{N} \right)^{1.2} \left(\frac{\phi_0}{10^{11} \text{ GeV}} \right)^{1.2}$$

Axino

Dark matter abundance

Axion Misalignment

$$\Omega_a \sim 0.1 \left(\frac{\sqrt{6}}{N} \right)^{1.2} \left(\frac{\phi_0}{10^{11} \text{ GeV}} \right)^{1.2}$$

Axino Flaton decay

$$\Omega_{\tilde{a}} \simeq 0.4 \left(\frac{\alpha_{\tilde{a}}}{10^{-1}} \right)^2 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right)^3 \left(\frac{10 \text{ GeV}}{T_d} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

Dark matter abundance

Axion Misalignment

$$\Omega_a \sim 0.1 \left(\frac{\sqrt{6}}{N} \right)^{1.2} \left(\frac{\phi_0}{10^{11} \text{ GeV}} \right)^{1.2}$$

Axino Flaton decay

$$\Omega_{\tilde{a}} \simeq 0.4 \left(\frac{\alpha_{\tilde{a}}}{10^{-1}} \right)^2 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right)^3 \left(\frac{10 \text{ GeV}}{T_d} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

Thermal NLSP decay

$$\Omega_{\tilde{a}} \sim 10^3 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

Dark matter abundance

Flaton decays late

$$T_d \sim 10 \text{ GeV} \left(\frac{|\mu|}{10^3 \text{ GeV}} \right)^2 \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)$$

Axion Misalignment

$$\Omega_a \sim 0.1 \left(\frac{\sqrt{6}}{N} \right)^{1.2} \left(\frac{\phi_0}{10^{11} \text{ GeV}} \right)^{1.2}$$

Axino Flaton decay

$$\Omega_{\tilde{a}} \simeq 0.4 \left(\frac{\alpha_{\tilde{a}}}{10^{-1}} \right)^2 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right)^3 \left(\frac{10 \text{ GeV}}{T_d} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

Thermal NLSP decay

$$\Omega_{\tilde{a}} \sim 10^3 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

Dark matter abundance

Flaton decays late

$$T_d \sim 10 \text{ GeV} \left(\frac{|\mu|}{10^3 \text{ GeV}} \right)^2 \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)$$

Axion Misalignment

$$\Omega_a \sim 0.1 \left(\frac{\sqrt{6}}{N} \right)^{1.2} \left(\frac{\phi_0}{10^{11} \text{ GeV}} \right)^{1.2} \times \begin{cases} 1 & \text{for } T_d \gg 1 \text{ GeV} \\ \left(\frac{T_d}{1 \text{ GeV}} \right)^2 & \text{for } T_d \ll 1 \text{ GeV} \end{cases}$$

Axino Flaton decay

$$\Omega_{\tilde{a}} \simeq 0.4 \left(\frac{\alpha_{\tilde{a}}}{10^{-1}} \right)^2 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right)^3 \left(\frac{10 \text{ GeV}}{T_d} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

Thermal NLSP decay

$$\Omega_{\tilde{a}} \sim 10^3 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

Dark matter abundance

Flaton decays late

$$T_d \sim 10 \text{ GeV} \left(\frac{|\mu|}{10^3 \text{ GeV}} \right)^2 \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)$$

Axion Misalignment

$$\Omega_a \sim 0.1 \left(\frac{\sqrt{6}}{N} \right)^{1.2} \left(\frac{\phi_0}{10^{11} \text{ GeV}} \right)^{1.2} \times \begin{cases} 1 & \text{for } T_d \gg 1 \text{ GeV} \\ \left(\frac{T_d}{1 \text{ GeV}} \right)^2 & \text{for } T_d \ll 1 \text{ GeV} \end{cases}$$

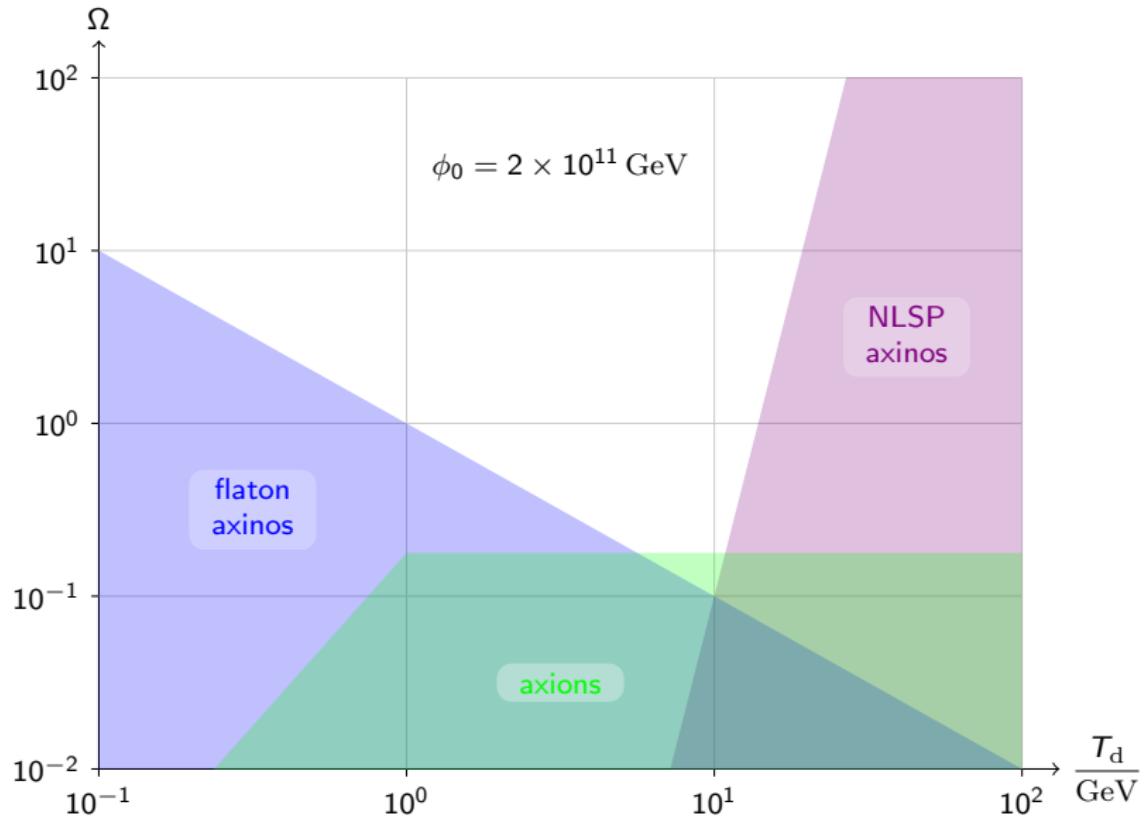
Axino Flaton decay

$$\Omega_{\tilde{a}} \simeq 0.4 \left(\frac{\alpha_{\tilde{a}}}{10^{-1}} \right)^2 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right)^3 \left(\frac{10 \text{ GeV}}{T_d} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2$$

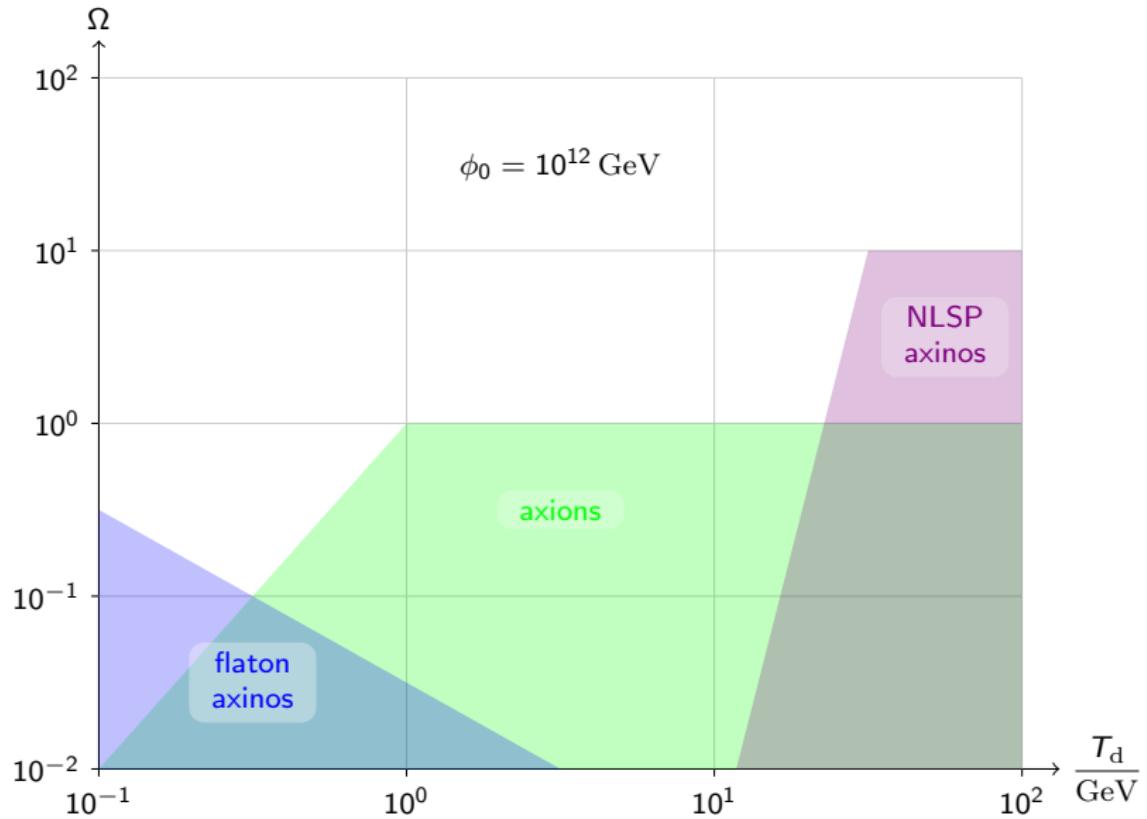
Thermal NLSP decay

$$\Omega_{\tilde{a}} \sim 10^3 \left(\frac{m_{\tilde{a}}}{1 \text{ GeV}} \right) \left(\frac{10^{11} \text{ GeV}}{\phi_0} \right)^2 \times \begin{cases} 1 & \text{for } T_d \gg \frac{m_N}{7} \\ \left(\frac{7 T_d}{m_N} \right)^7 & \text{for } T_d \ll \frac{m_N}{7} \end{cases}$$

Dark matter composition

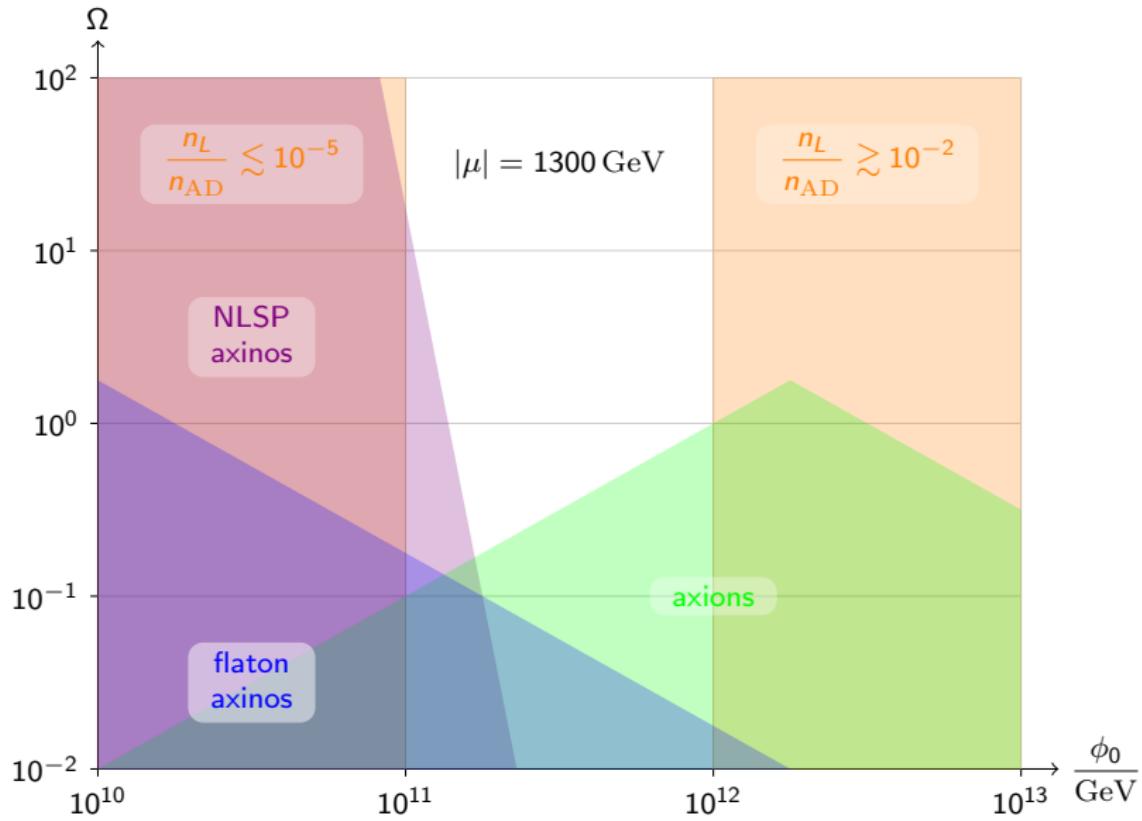


Dark matter composition



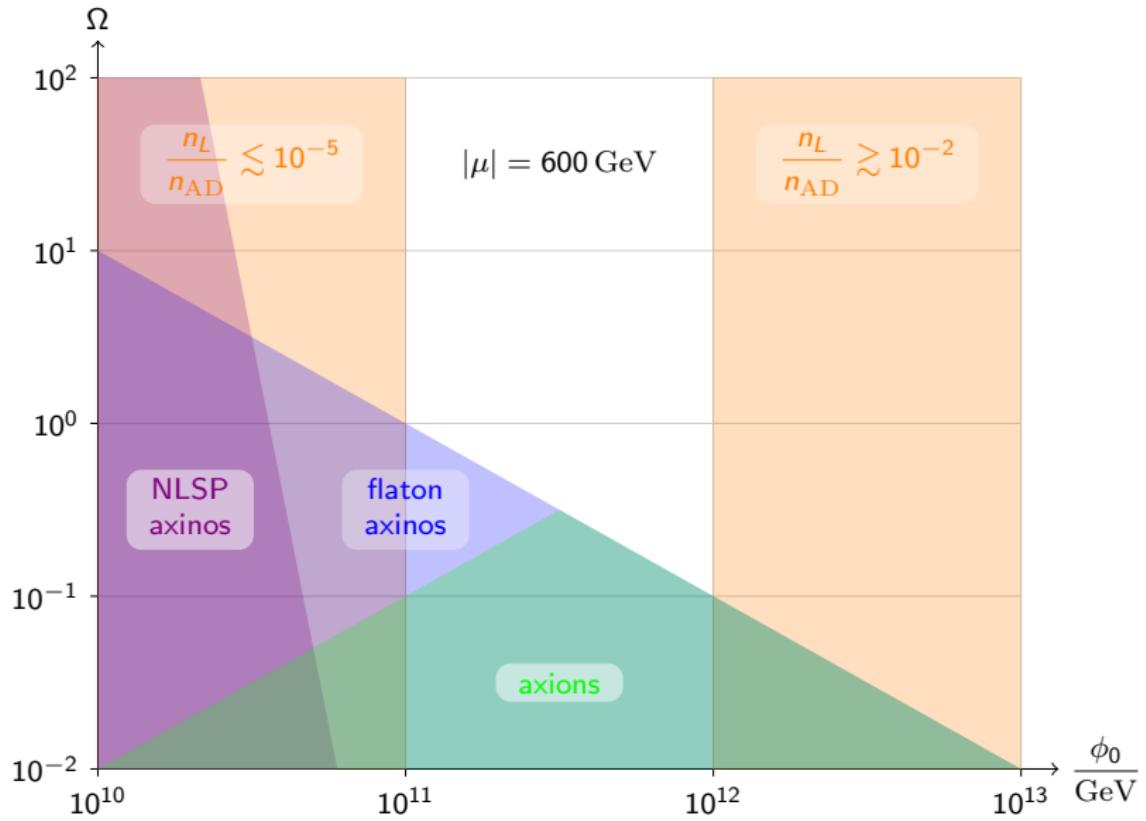
$\phi_0 = 2 \times 10^{11} \text{ GeV}$

Dark matter composition



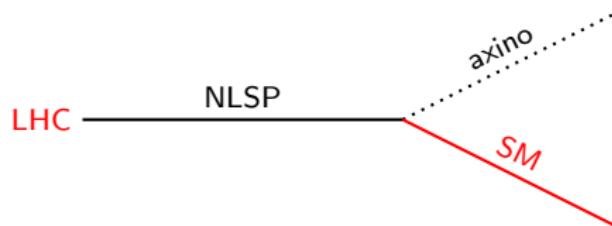
$|\mu| = 600 \text{ GeV}$

Dark matter composition



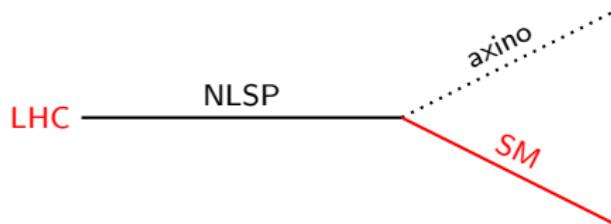
Axino LHC signal

NLSPs produced by the LHC decay to axinos plus Standard Model particles



Axino LHC signal

NLSPs produced by the LHC decay to axinos plus Standard Model particles



with a decay length

$$\frac{1}{\Gamma_{N \rightarrow \tilde{a}}} \sim \frac{16\pi\phi_0^2}{m_N^3} \sim 100 \text{ m} \left(\frac{200 \text{ GeV}}{m_N} \right)^3 \left(\frac{\phi_0}{3 \times 10^{11} \text{ GeV}} \right)^2$$

and well constrained parameters

$$10^{11} \text{ GeV} \lesssim \phi_0 \lesssim 10^{12} \text{ GeV}$$

$$m_{\tilde{a}} \simeq 1 \text{ GeV}$$

Simple model

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \frac{1}{2} \lambda_\nu (L H_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

Simple model

MSSM

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \frac{1}{2} \lambda_\nu (L H_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

Simple model

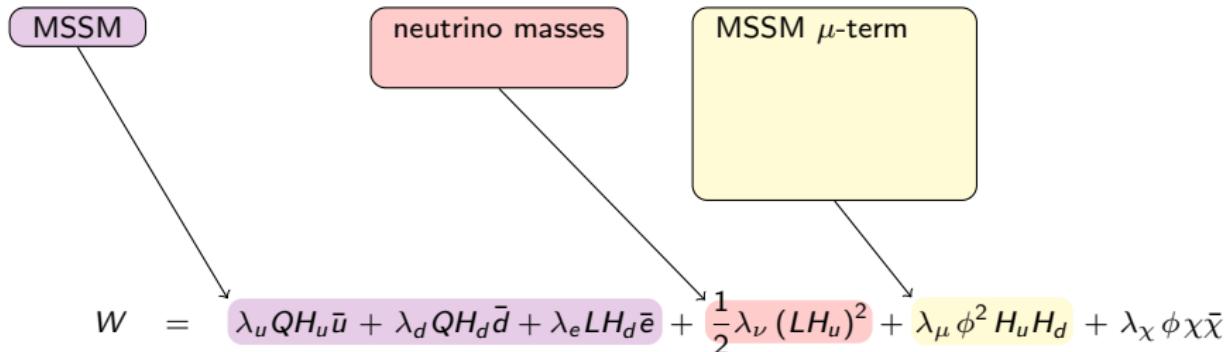
The diagram illustrates the decomposition of the scalar potential W into two parts: MSSM and neutrino masses.

The MSSM part is represented by the term $\lambda_u QH_u\bar{u} + \lambda_d QH_d\bar{d} + \lambda_e LH_d\bar{e}$.

The neutrino mass part is represented by the term $\frac{1}{2}\lambda_\nu (LH_u)^2 + \lambda_\mu \phi^2 H_uH_d + \lambda_\chi \phi\chi\bar{\chi}$.

$$W = \lambda_u QH_u\bar{u} + \lambda_d QH_d\bar{d} + \lambda_e LH_d\bar{e} + \frac{1}{2}\lambda_\nu (LH_u)^2 + \lambda_\mu \phi^2 H_uH_d + \lambda_\chi \phi\chi\bar{\chi}$$

Simple model



Simple model

$$W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \frac{1}{2} \lambda_\nu (L H_u)^2 + \lambda_\mu \phi^2 H_u H_d + \lambda_\chi \phi \chi \bar{\chi}$$

MSSM

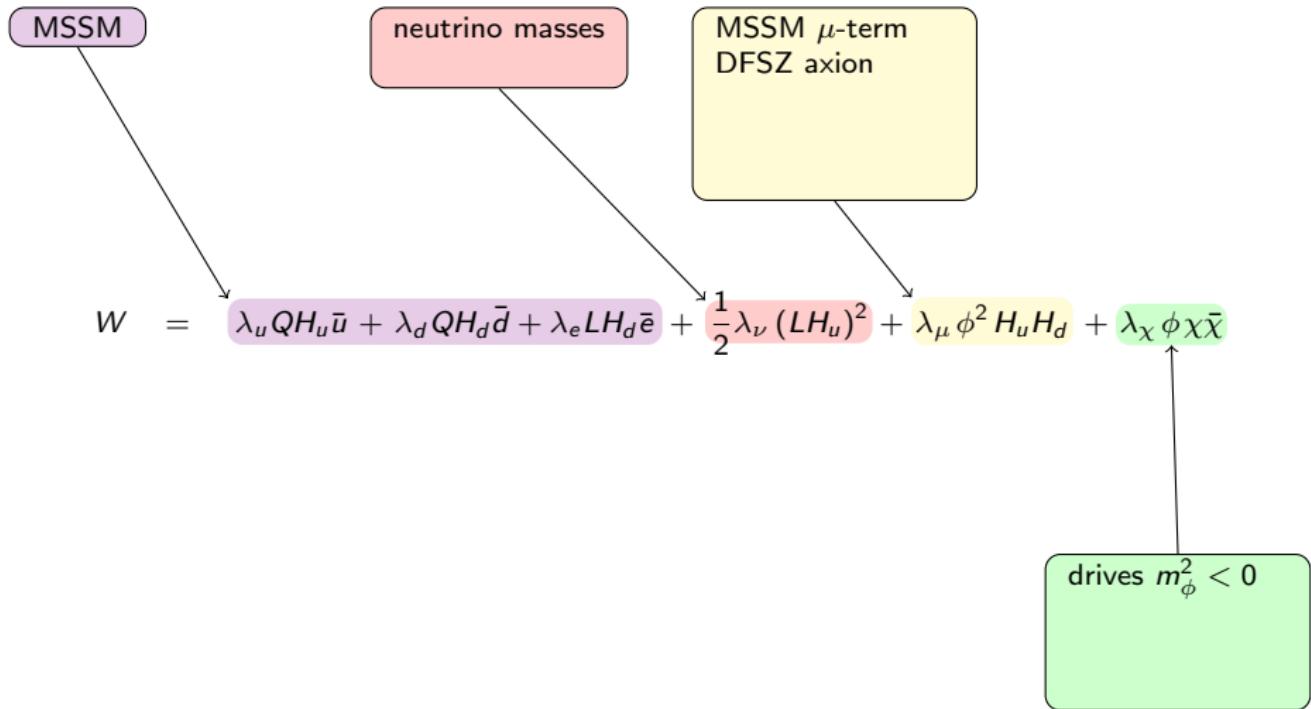
neutrino masses

MSSM μ -term

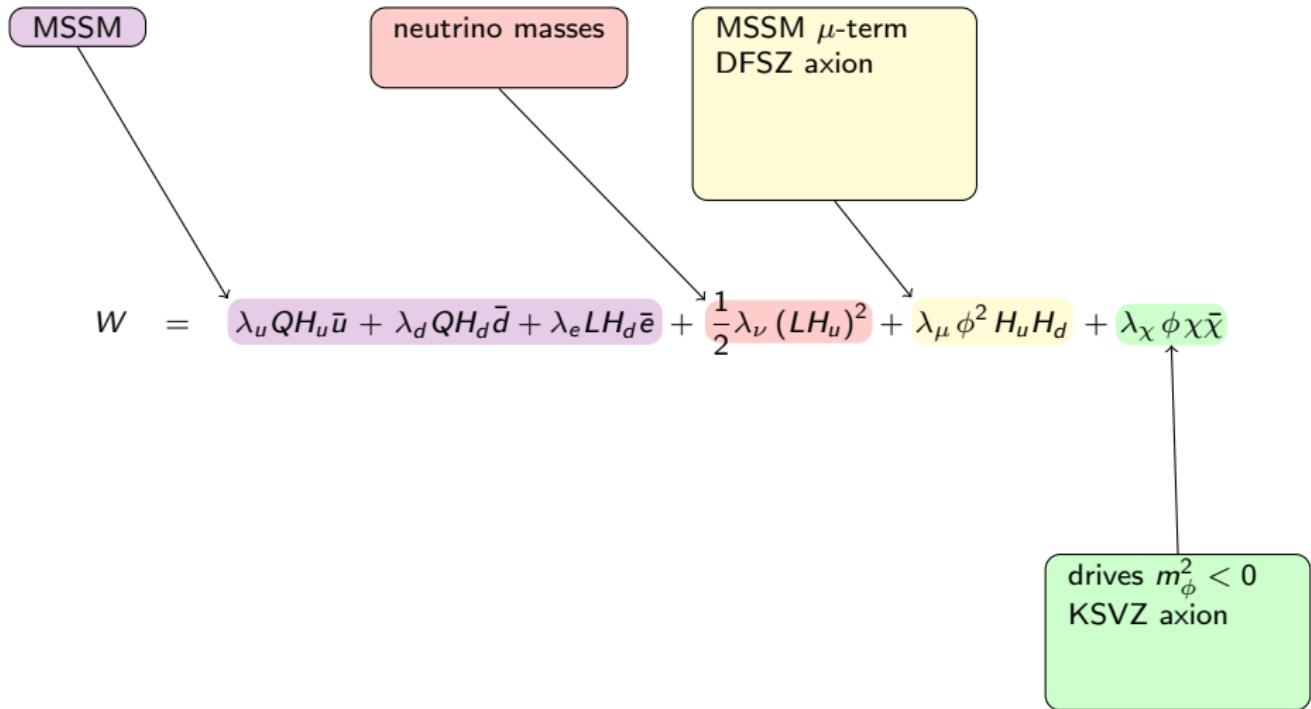
```
graph TD; MSSM[MSSM] --> W1[λu Q H_u bar{u}]; Neutrino[neutrino masses] --> W2[λd Q H_d bar{d}]; MuTerm[MSSM μ-term] --> W3[1/2 λν (L H_u)²]; W4[λμ φ² H_u H_d] --> W5[λχ φ χ bar{χ}]; W5 --> Drives[drives m_φ² < 0]
```

drives $m_\phi^2 < 0$

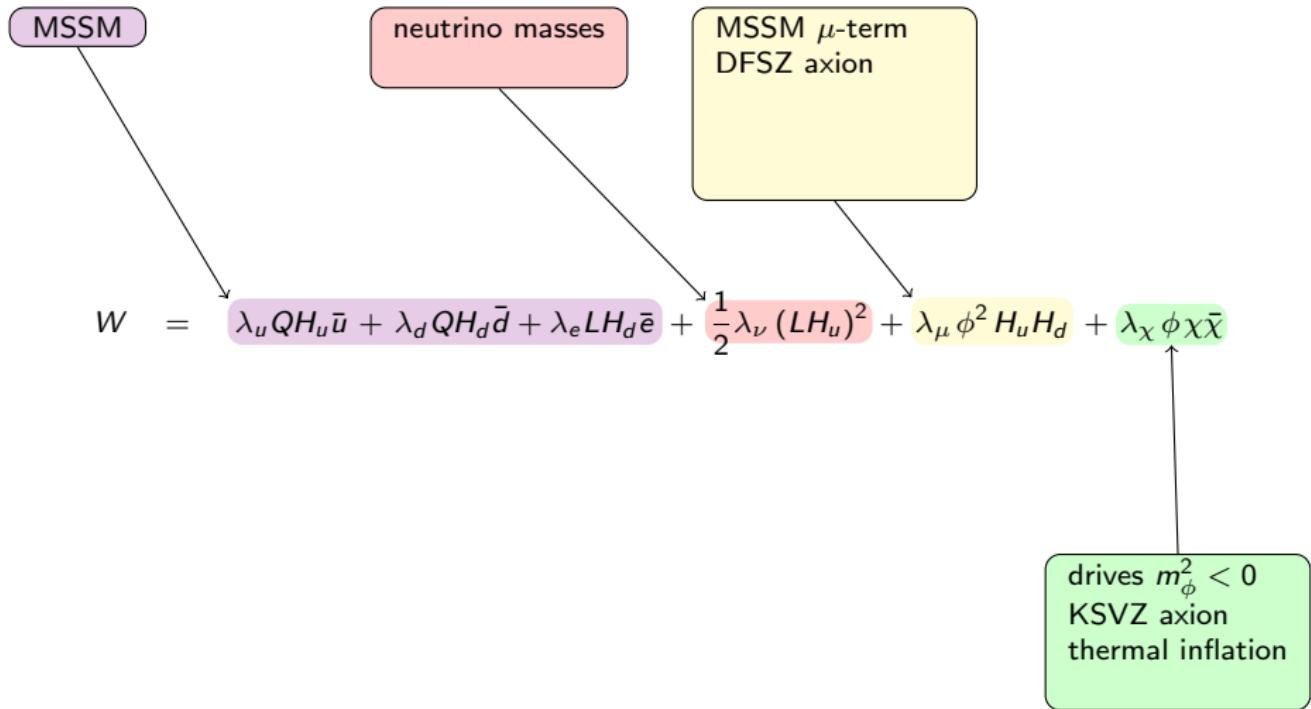
Simple model



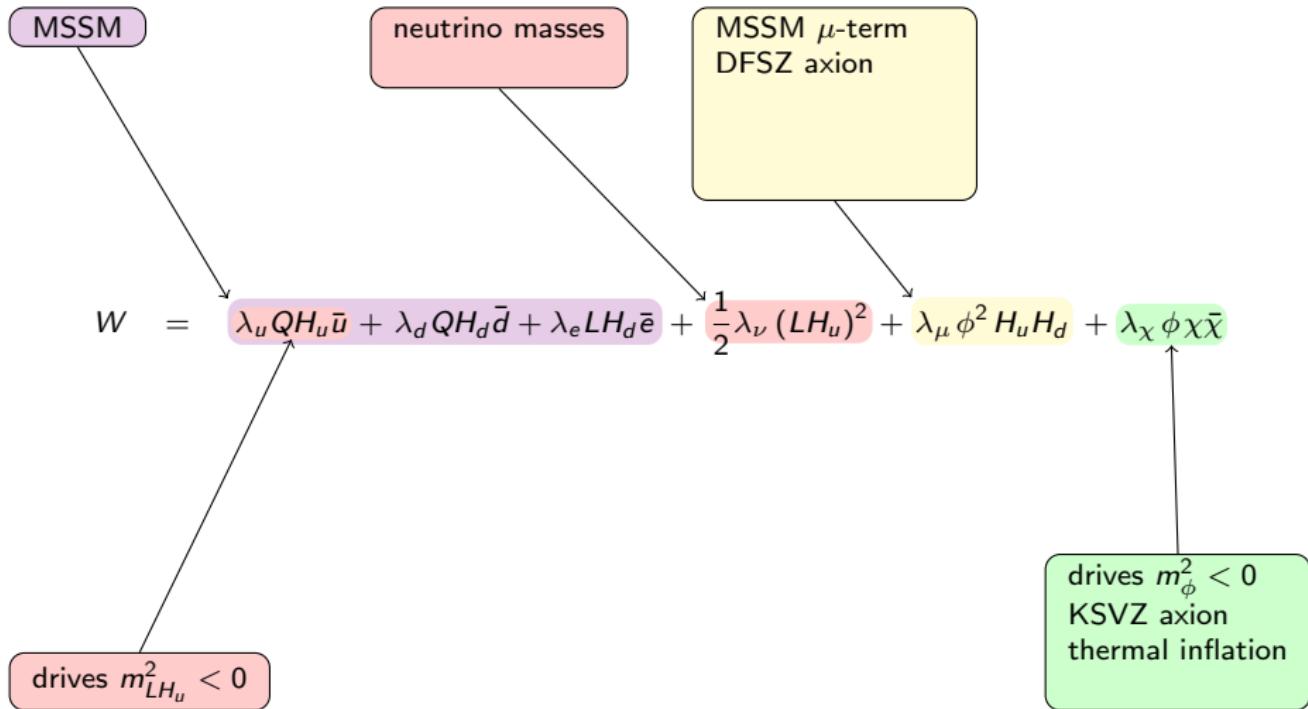
Simple model



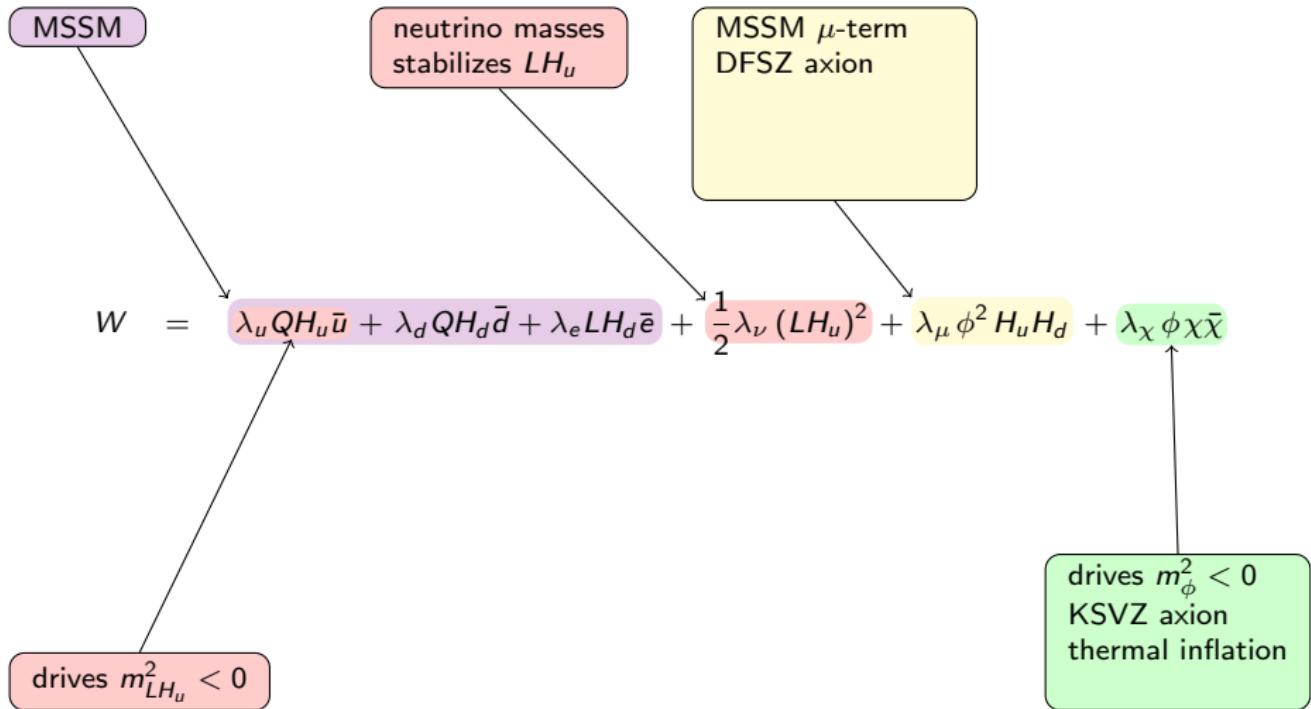
Simple model



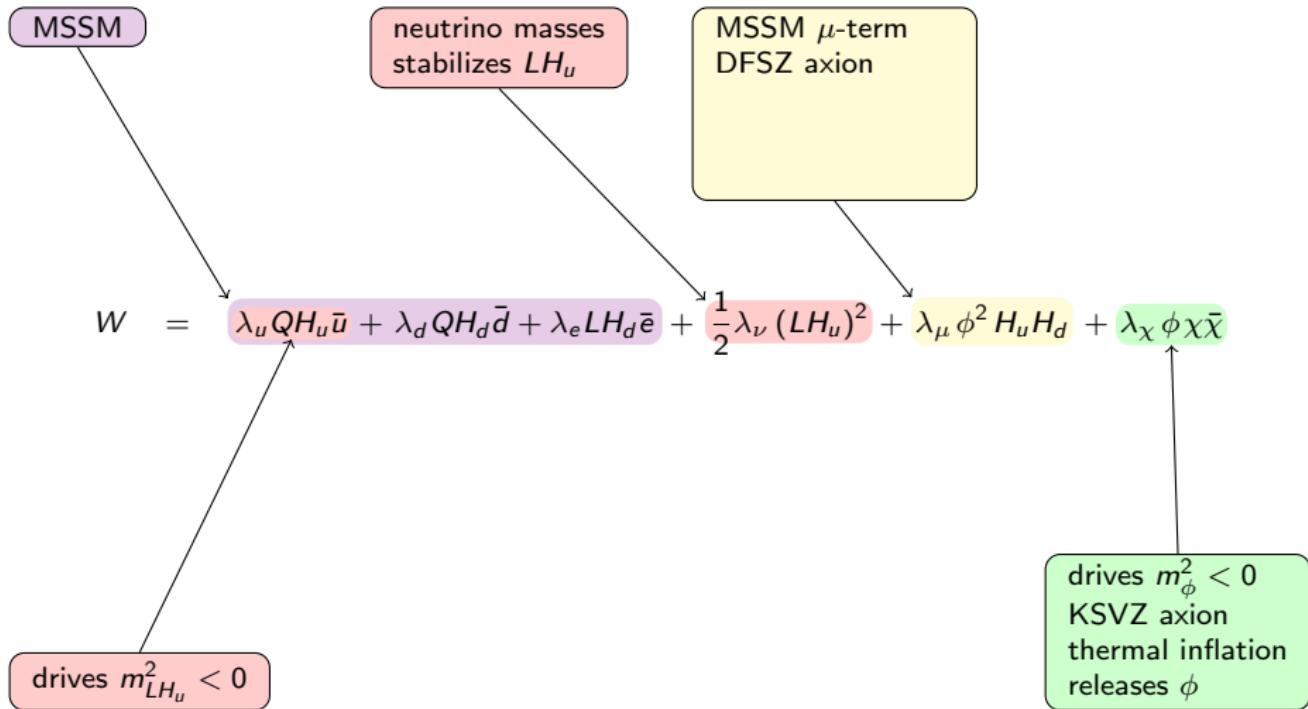
Simple model



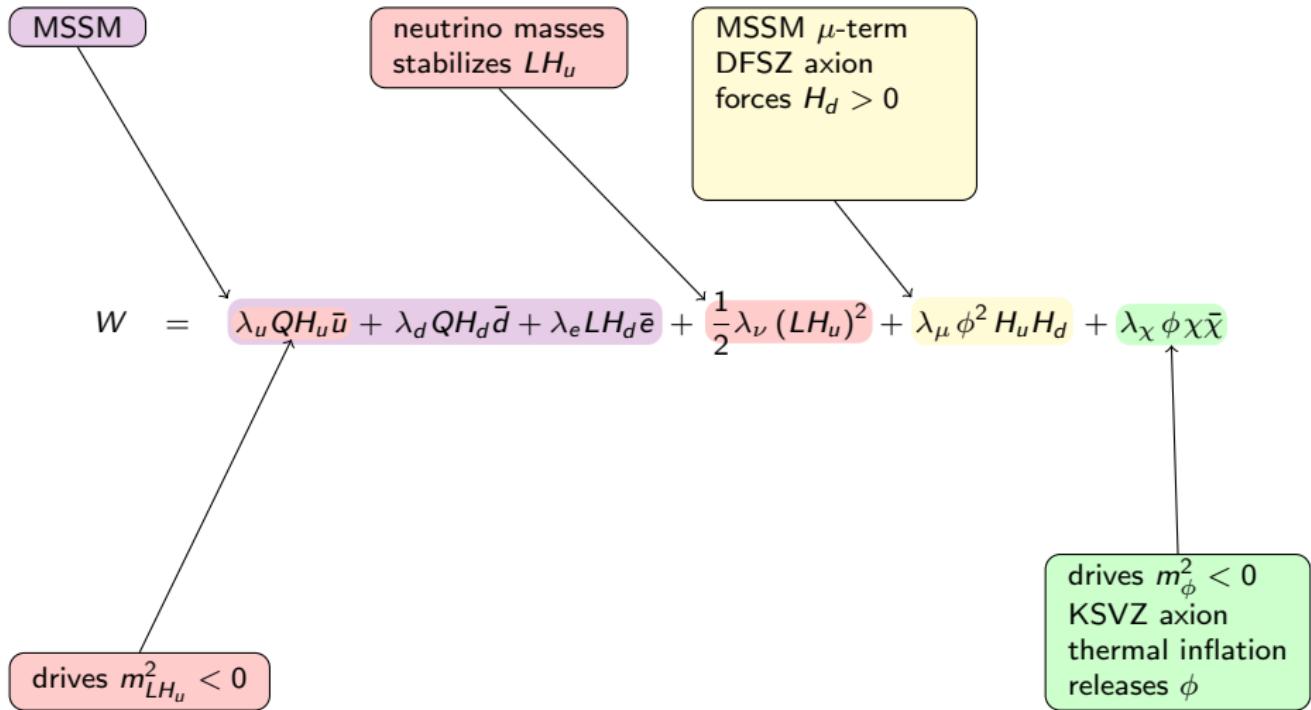
Simple model



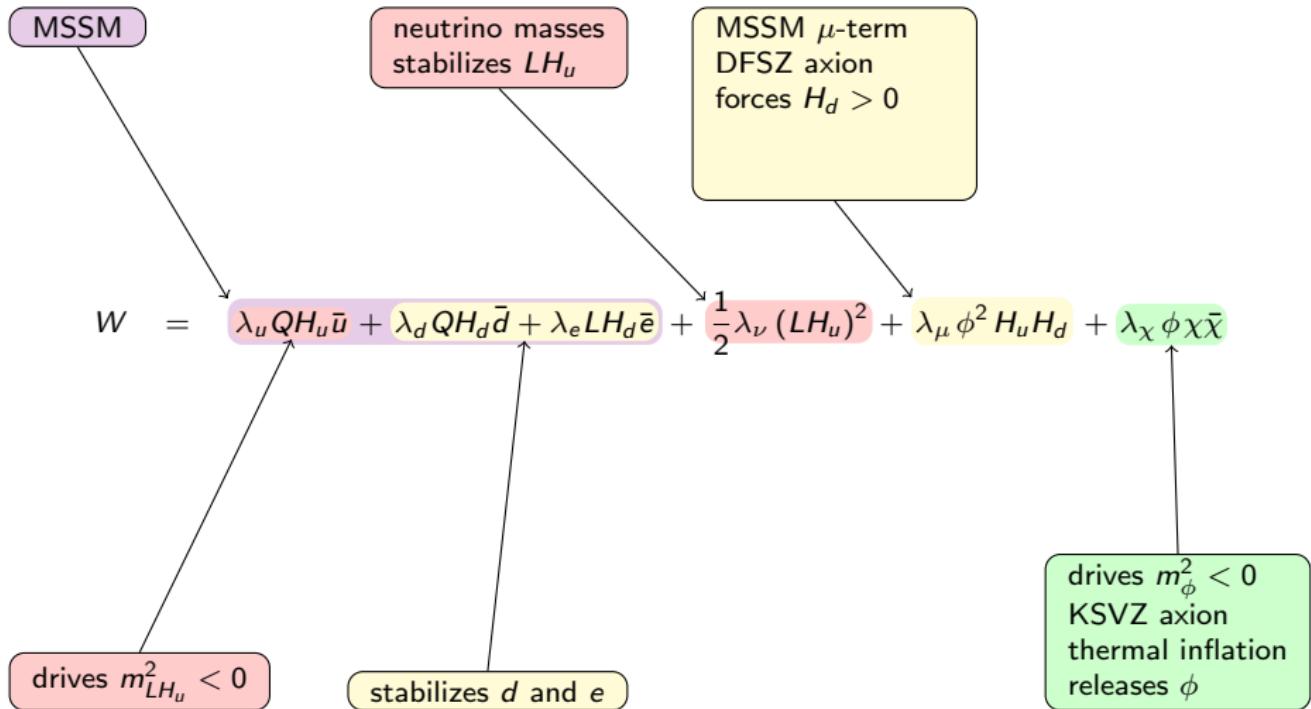
Simple model



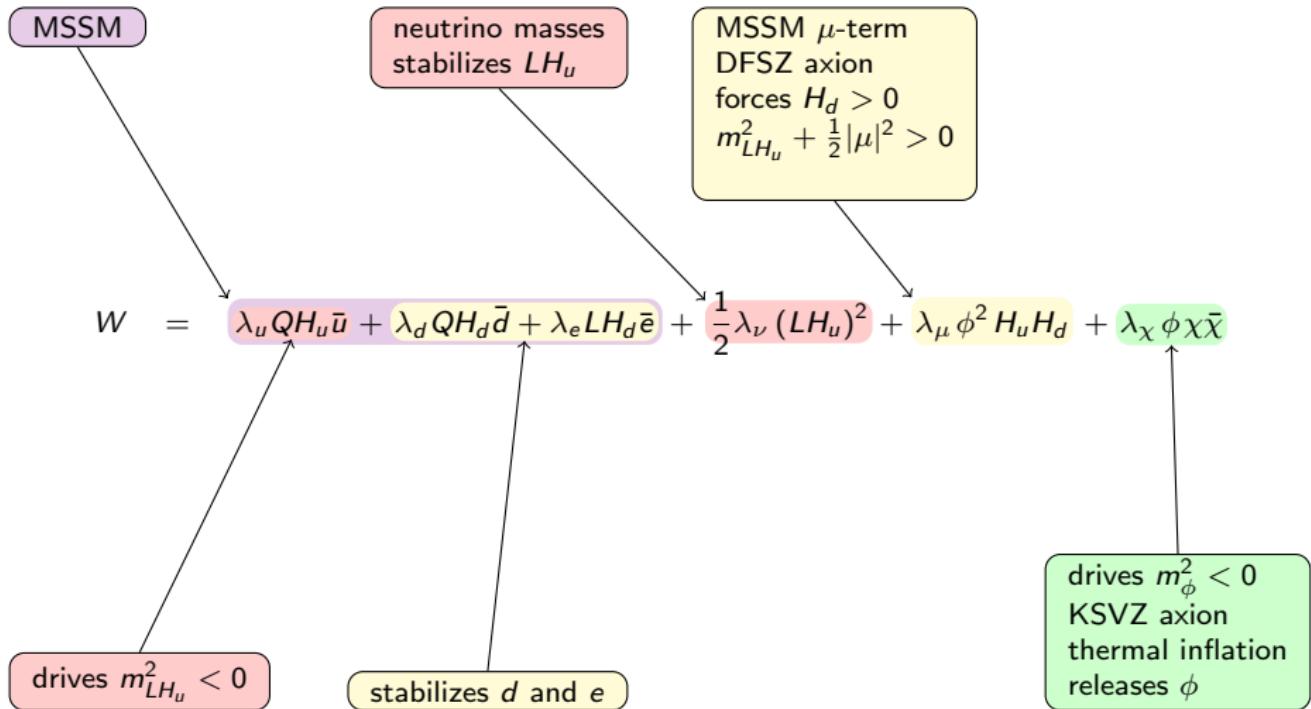
Simple model



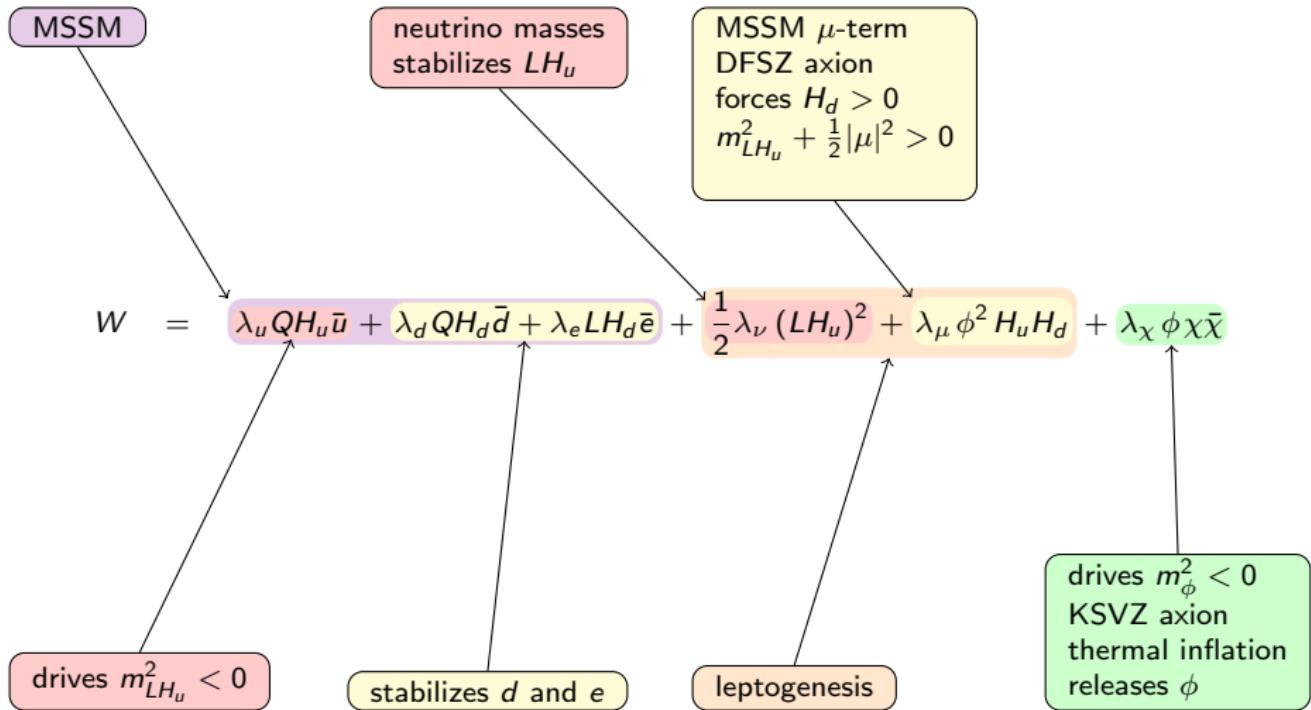
Simple model



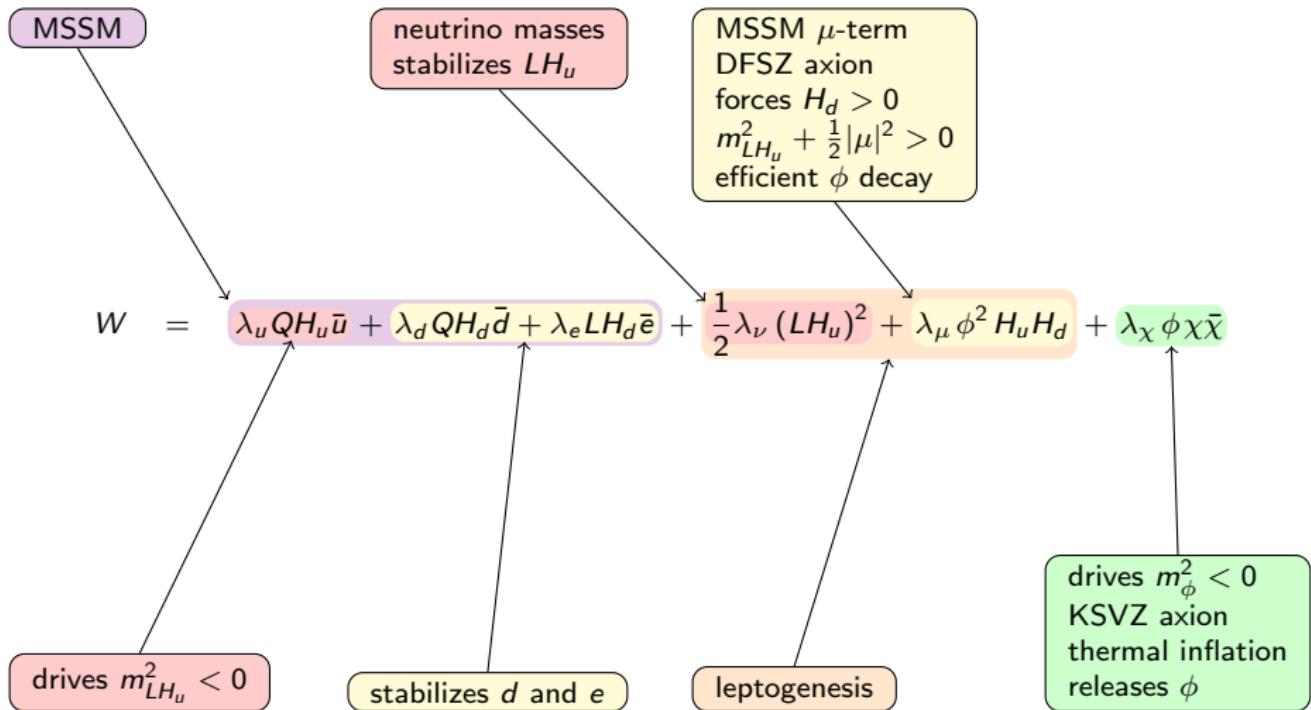
Simple model



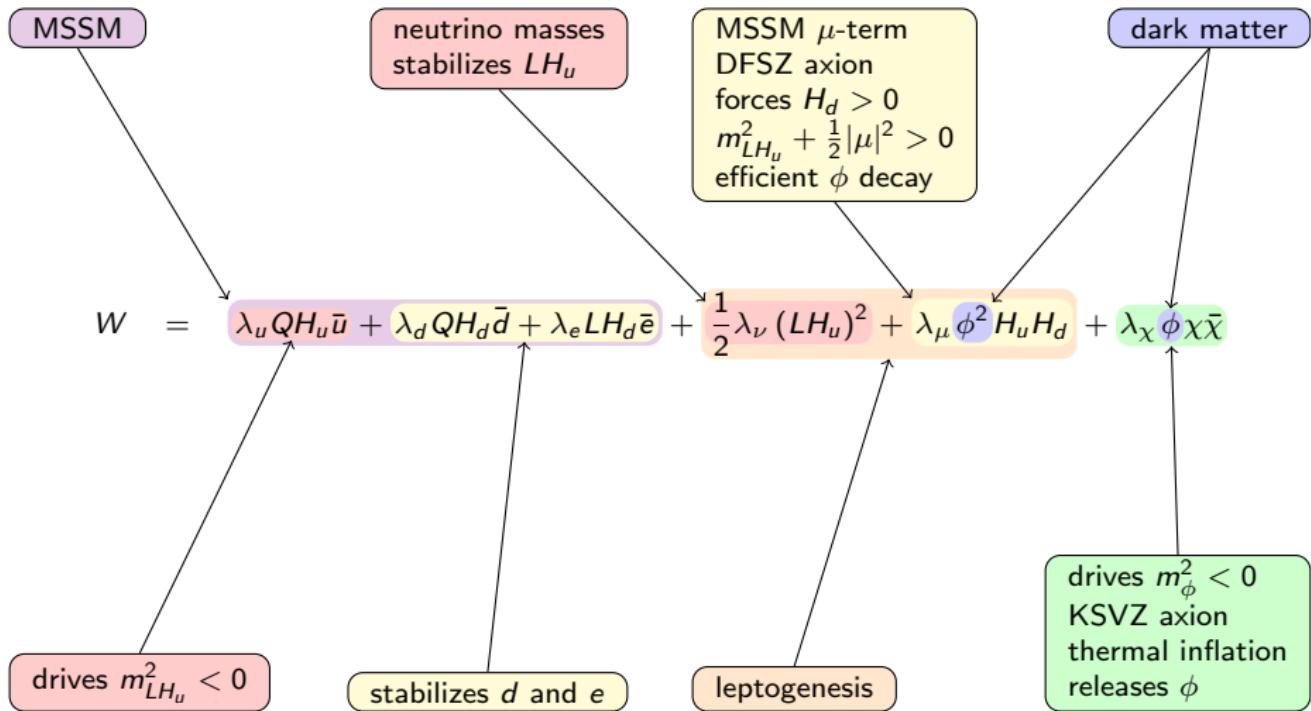
Simple model



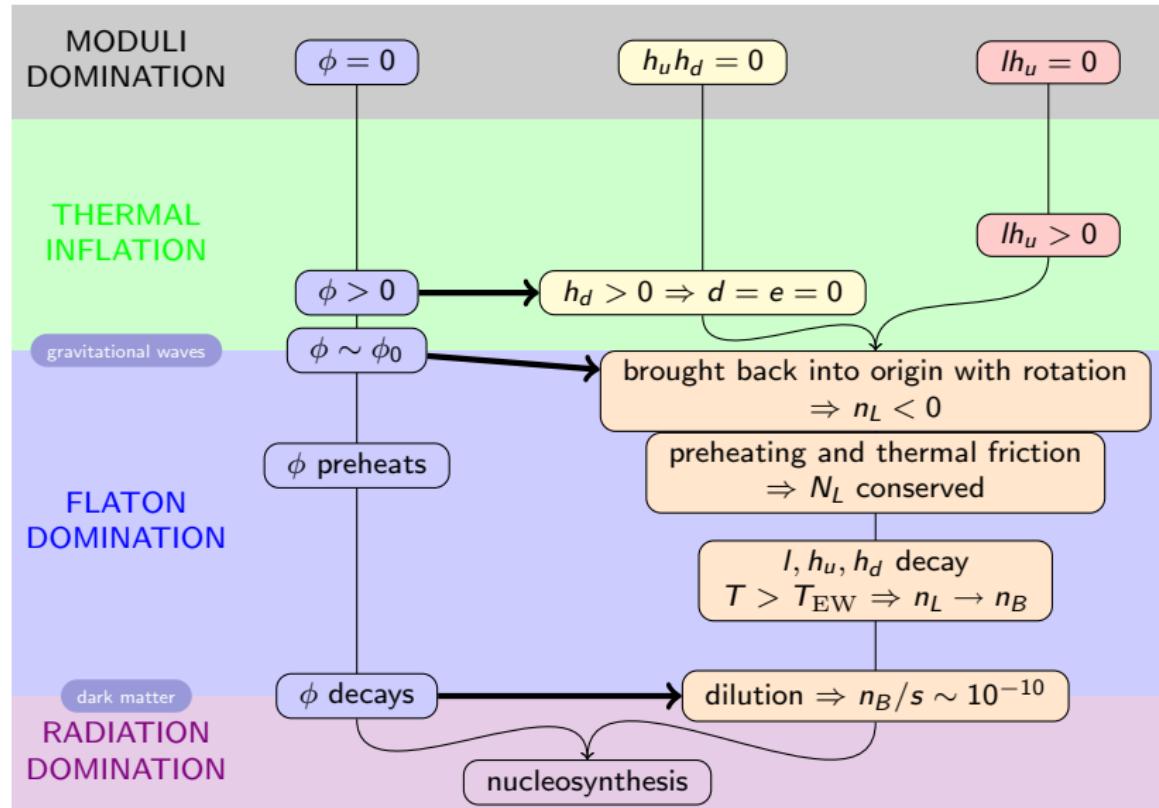
Simple model



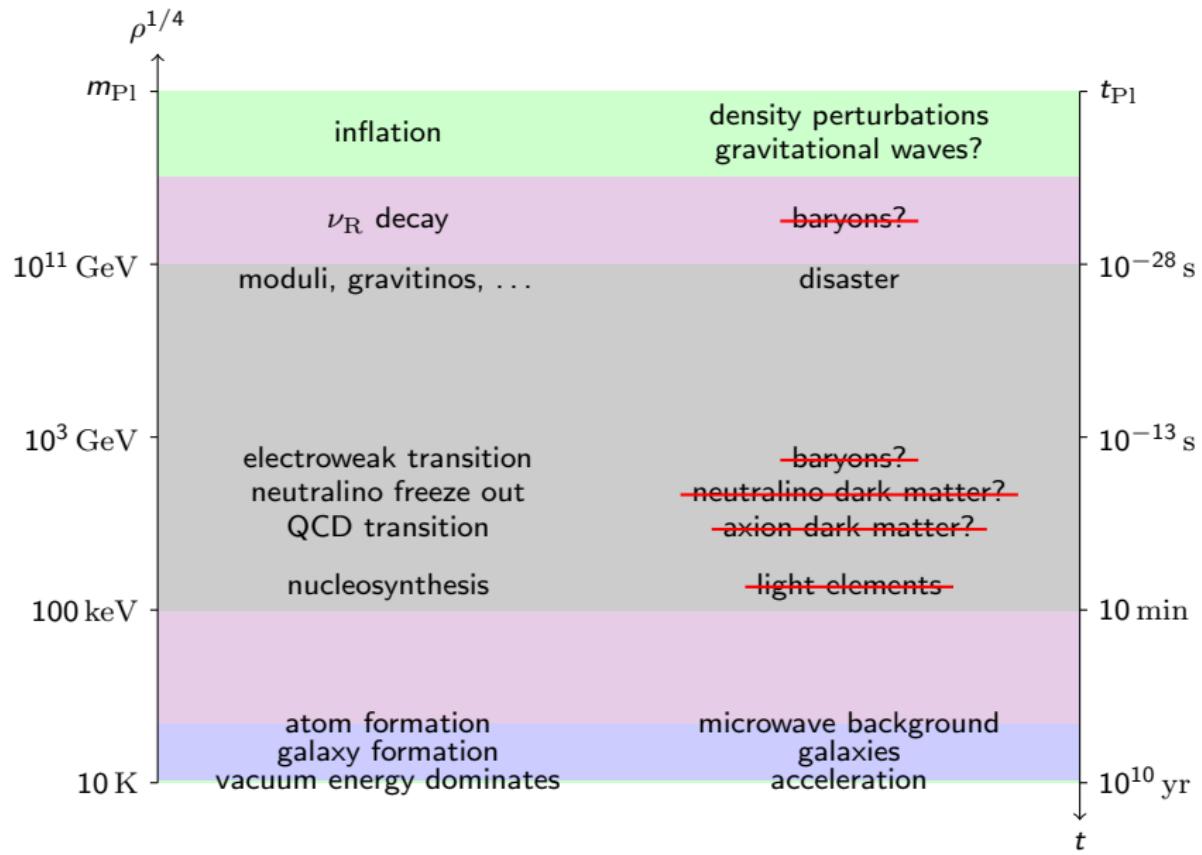
Simple model



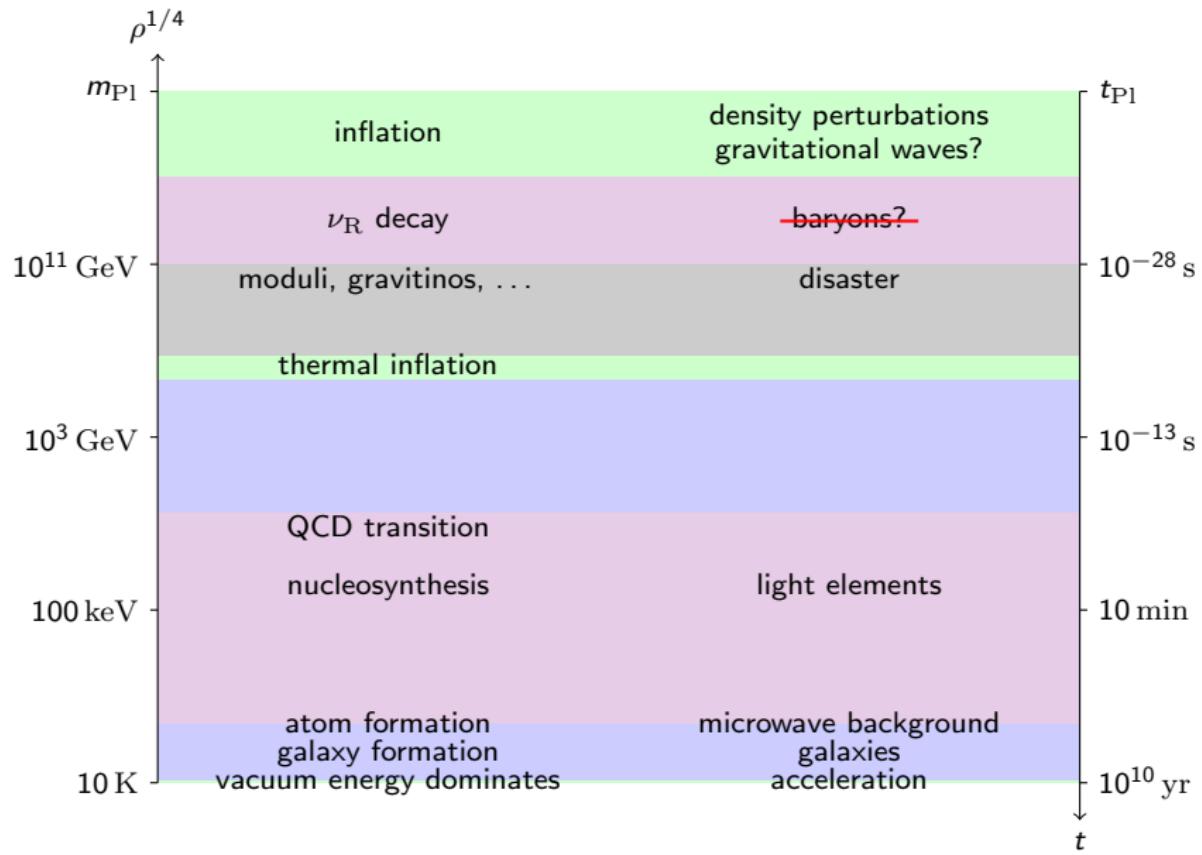
Rich cosmology



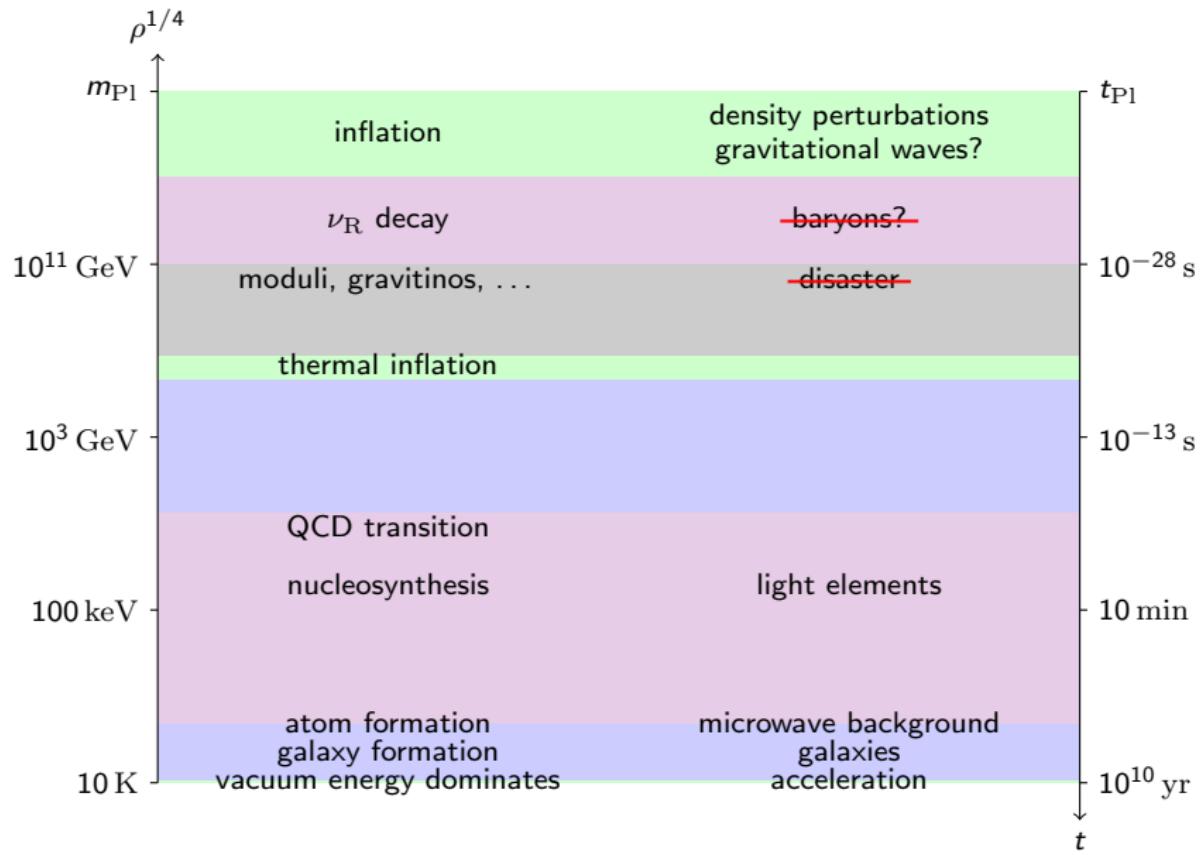
History of the observable universe



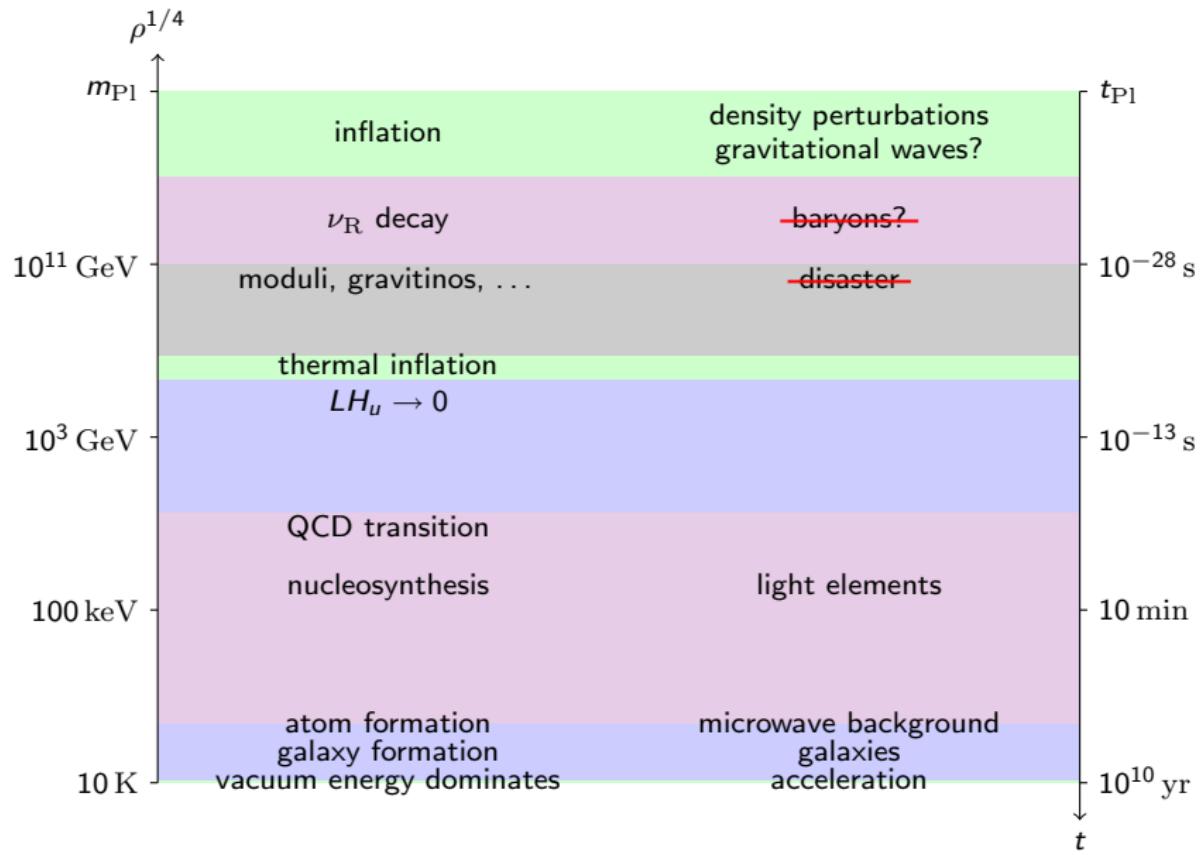
History of the observable universe



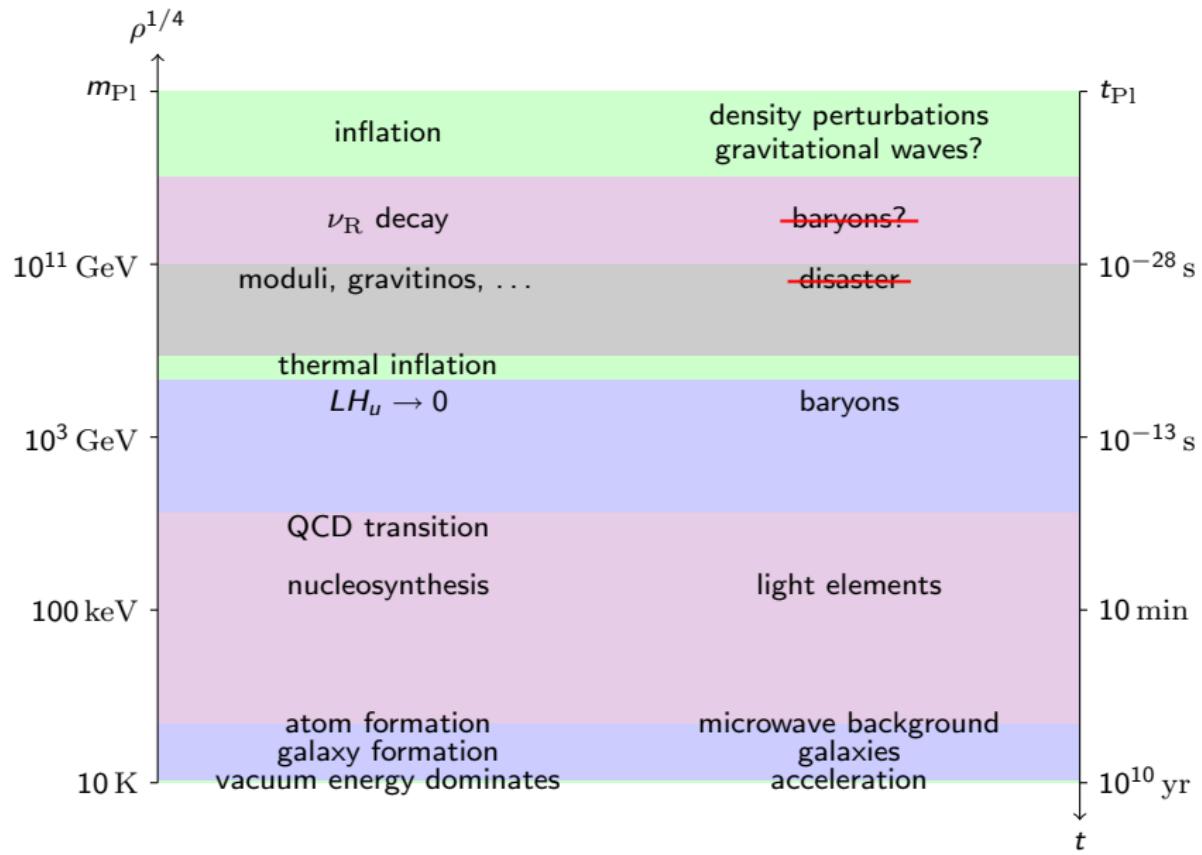
History of the observable universe



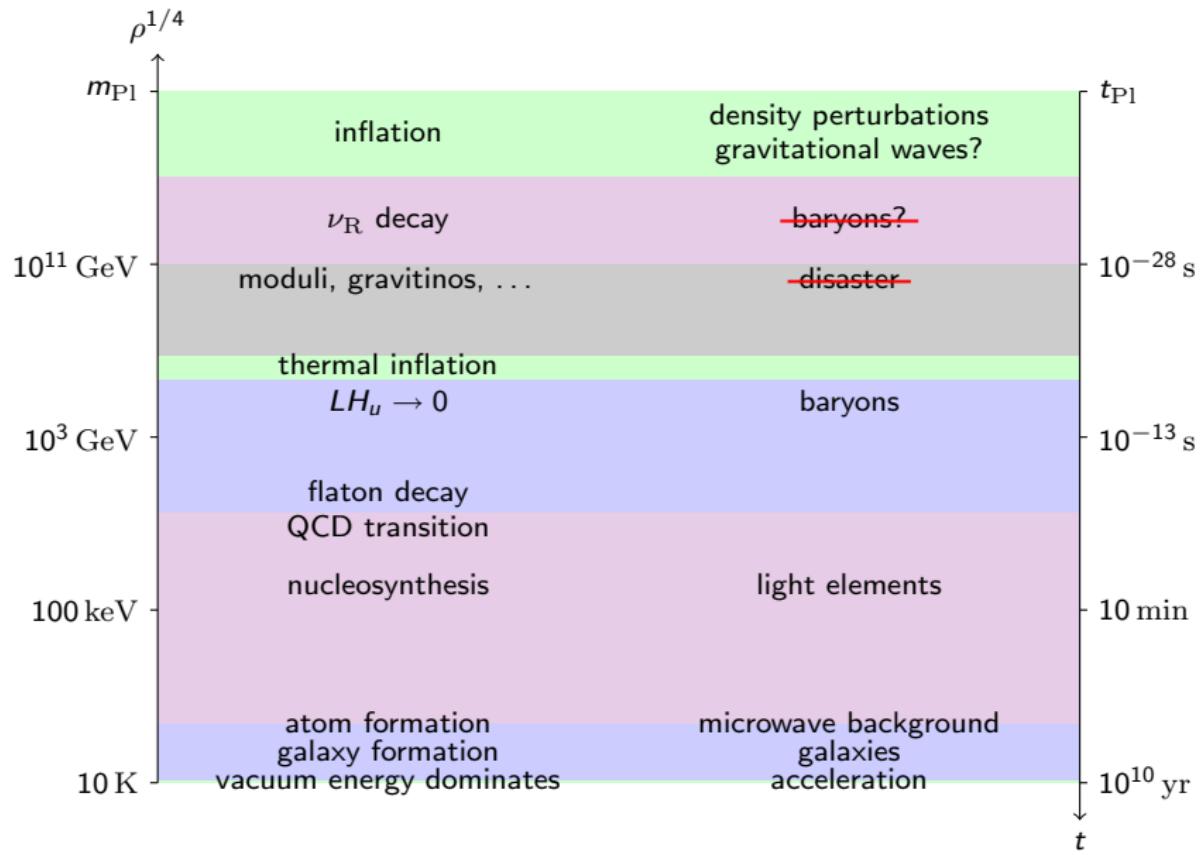
History of the observable universe



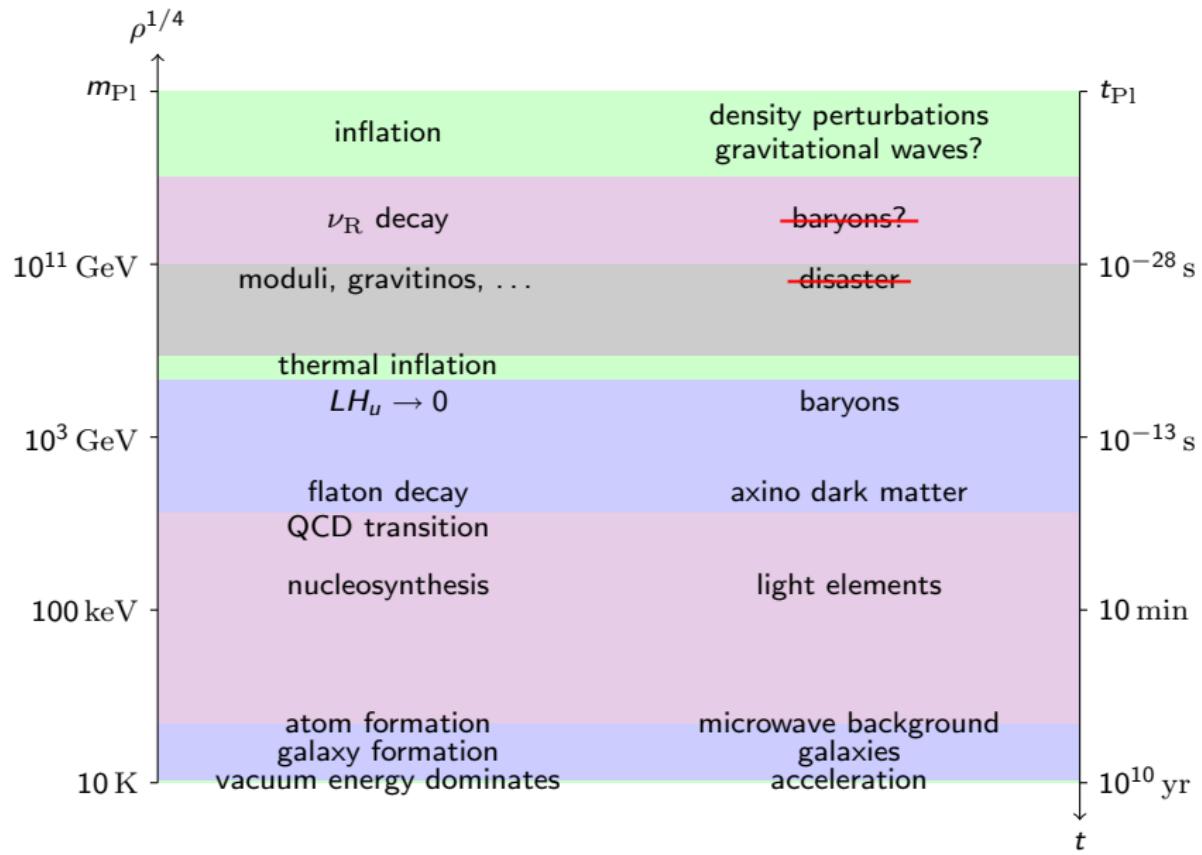
History of the observable universe



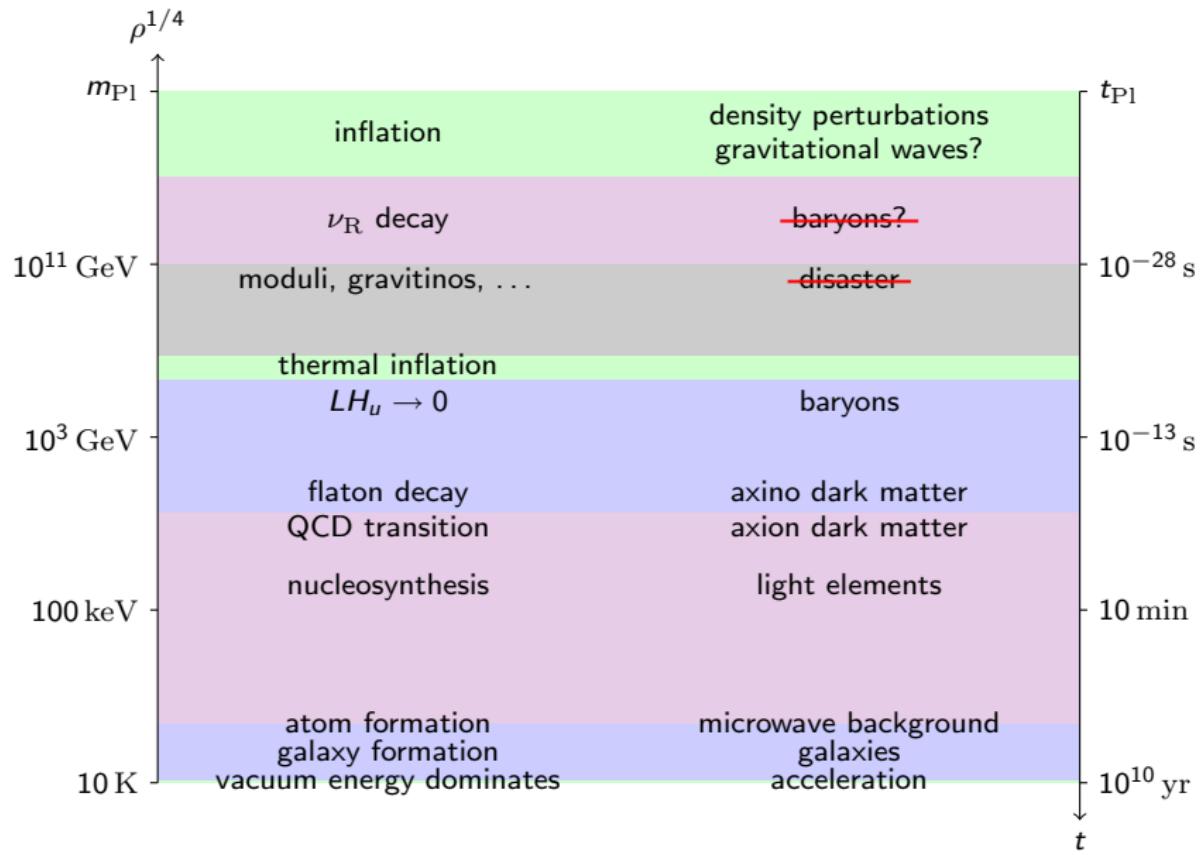
History of the observable universe



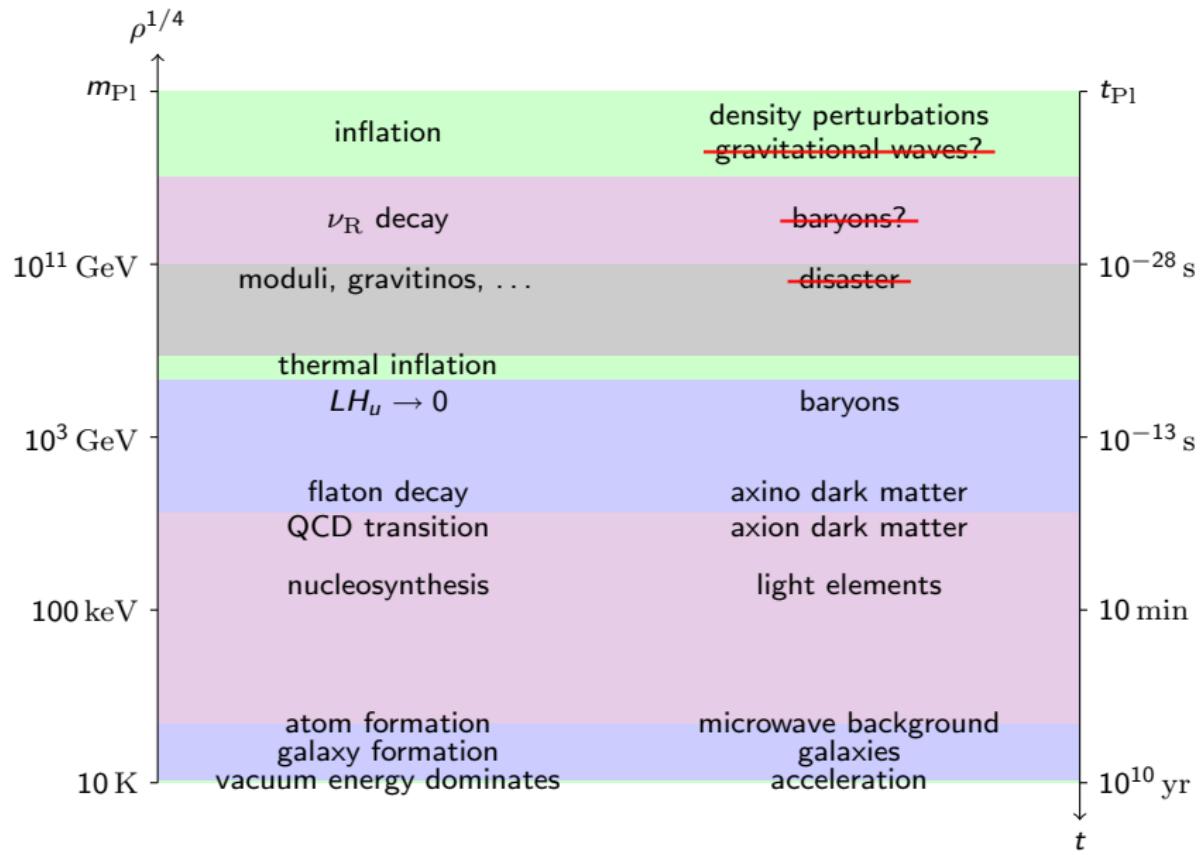
History of the observable universe



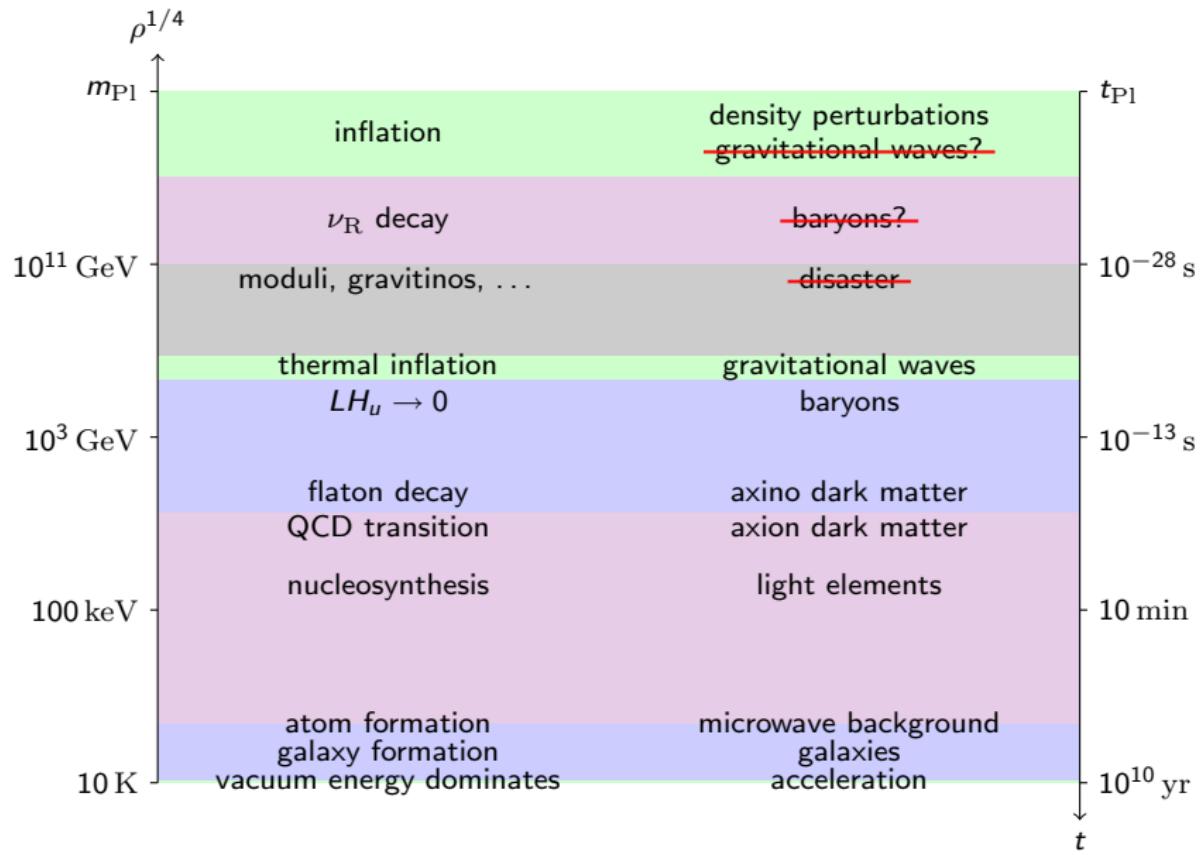
History of the observable universe



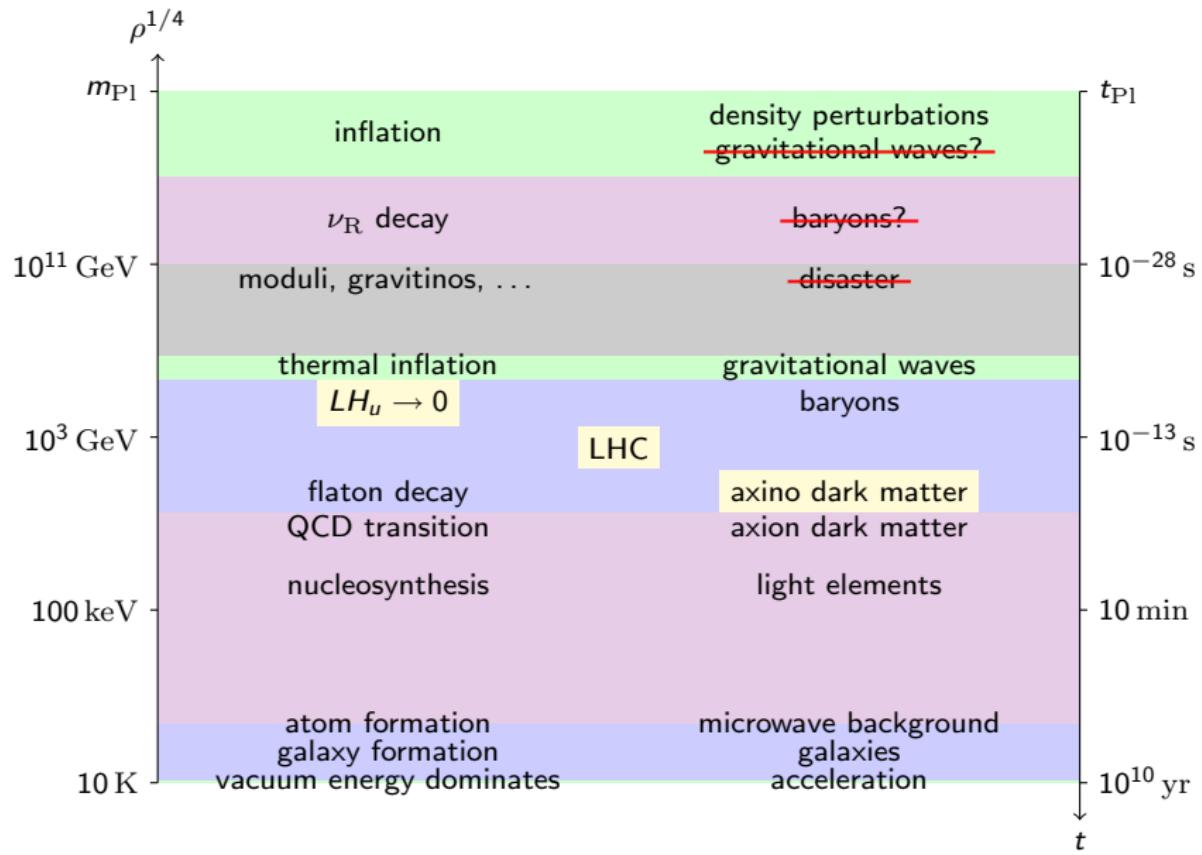
History of the observable universe



History of the observable universe



History of the observable universe



Introduction

History of the observable universe

Thermal inflation and gravitational waves

Moduli problem

Thermal inflation

First order phase transition

Gravitational waves

Baryogenesis

Superpotential

Key assumption

Reduction

Potential

Cosmology

Numerical simulation

Lepton number

Baryon asymmetry

Dark matter

Candidates

Abundance

Composition

LHC signal

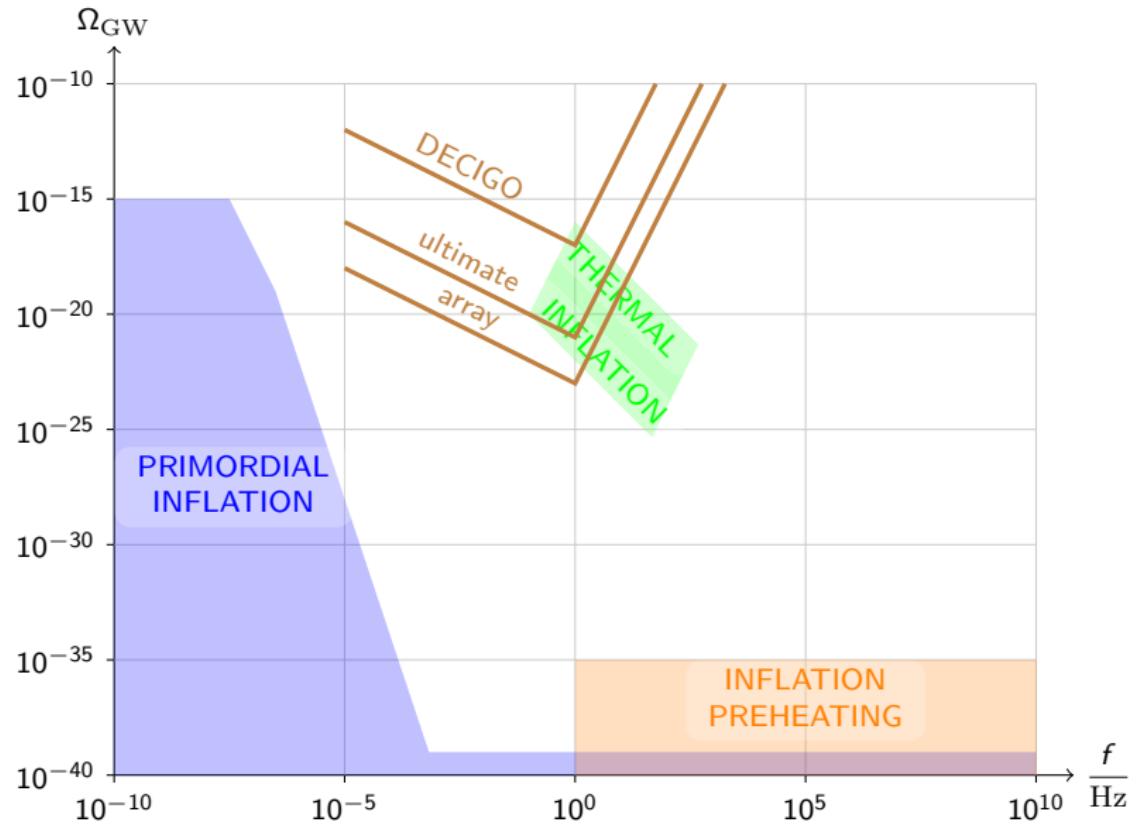
Summary

Simple model

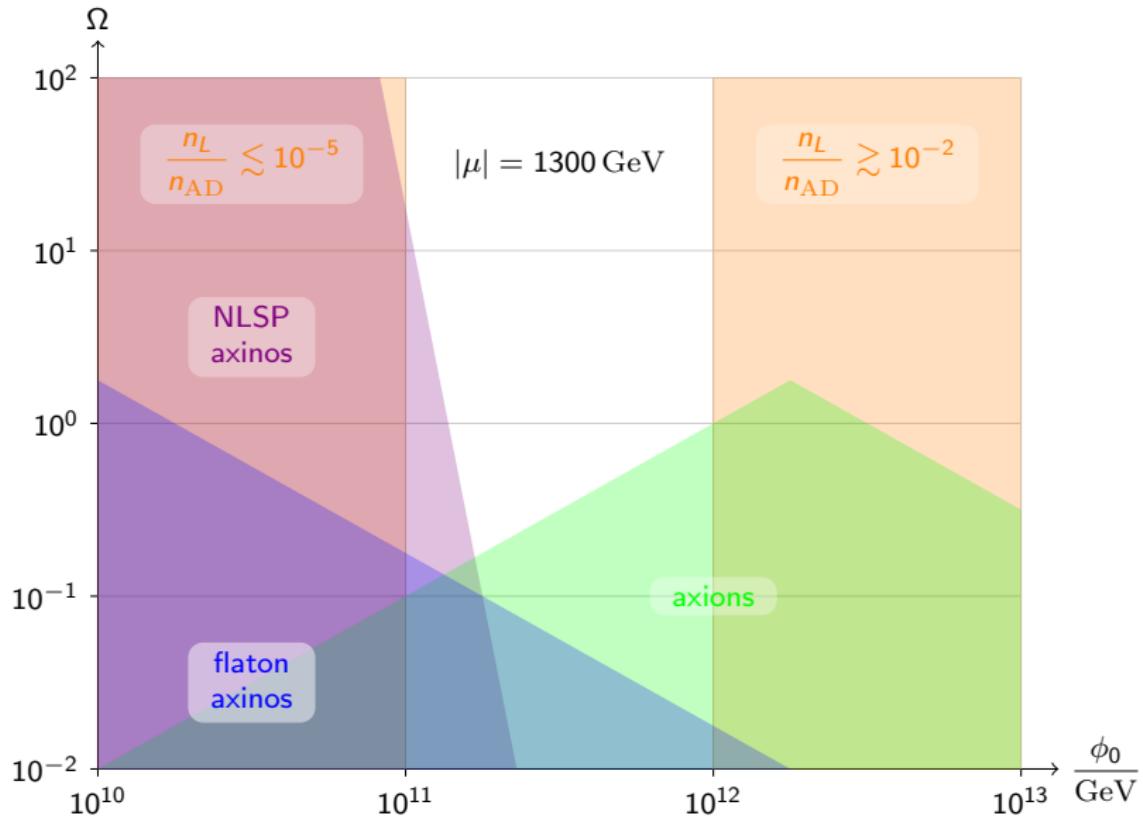
Rich cosmology

History of the observable universe

Gravitational waves

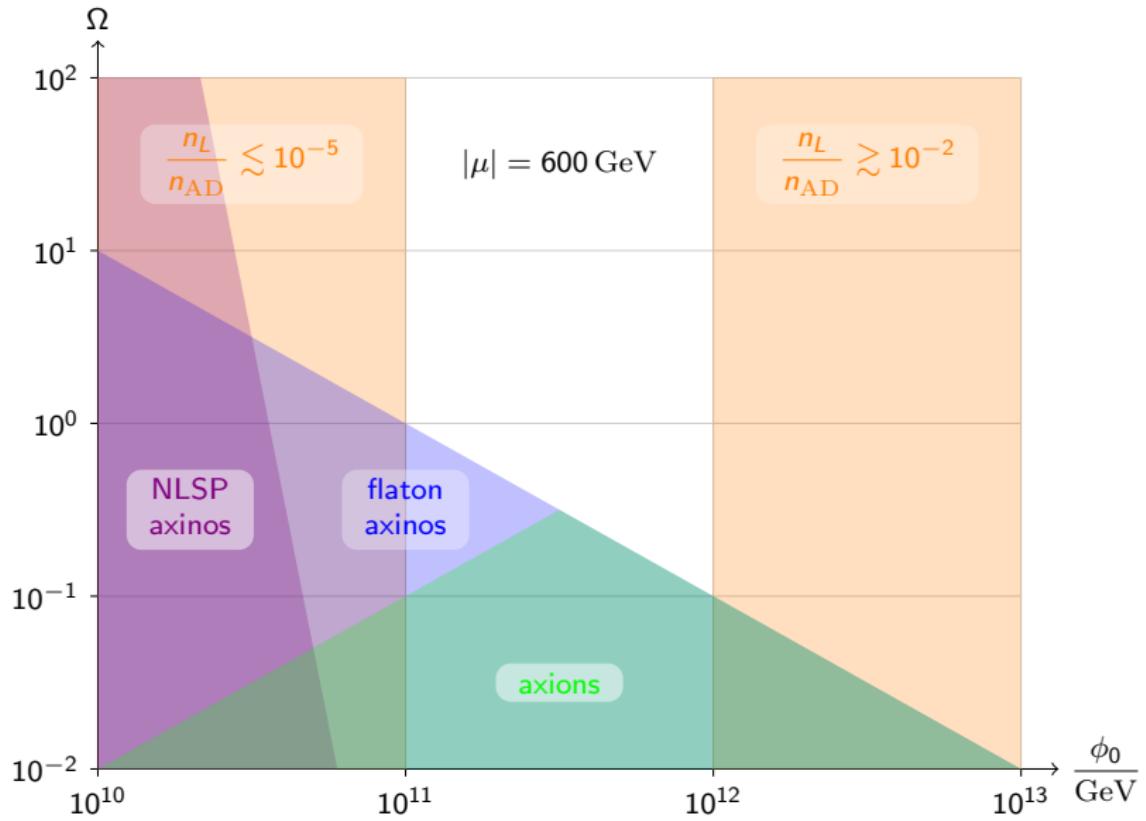


Dark matter composition



$|\mu| = 600 \text{ GeV}$

Dark matter composition



$|\mu| = 1300 \text{ GeV}$

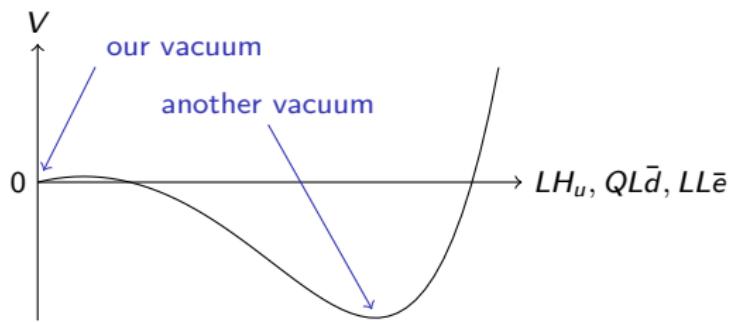
Key assumption

$$m_{LH_u}^2 = \frac{1}{2} (m_L^2 + m_{H_u}^2) < 0$$

Implies a dangerous non-MSSM vacuum with $LH_u \sim (10^9 \text{ GeV})^2$ and

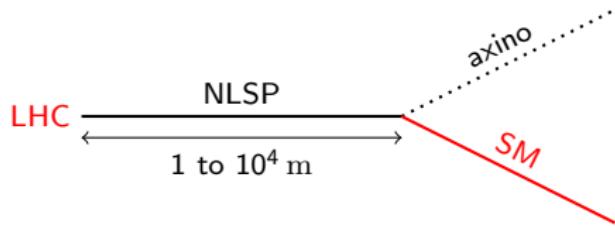
$$\lambda_d QL\bar{d} + \lambda_e LL\bar{e} = -\mu LH_u$$

eliminating the μ -term contribution to LH_u 's mass squared.



Axino LHC signal

NLSPs produced by the LHC decay to axinos plus Standard Model particles



with a decay length

$$\frac{1}{\Gamma_{N \rightarrow \tilde{a}}} \sim \frac{16\pi\phi_0^2}{m_N^3} \sim 100 \text{ m} \left(\frac{200 \text{ GeV}}{m_N} \right)^3 \left(\frac{\phi_0}{3 \times 10^{11} \text{ GeV}} \right)^2$$

and well constrained parameters

$$10^{11} \text{ GeV} \lesssim \phi_0 \lesssim 10^{12} \text{ GeV}$$

$$m_{\tilde{a}} \simeq 1 \text{ GeV}$$

