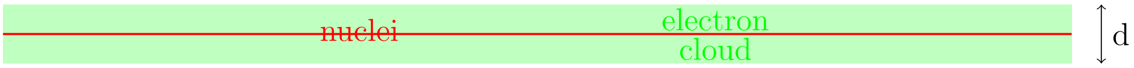
Homework 6 - Fundamental interactions

Q6.1. Model the surface of an object by a infinitely thin sheet, of surface charge density σ, representing the atomic nuclei, surrounded by a uniform electron cloud, of width d and charge density $-\sigma/d$, as shown in the figure.



Describe the interaction between two such surfaces.

A6.1. In this effectively one dimensional problem, the interactions will not vary with distance, so we only need to consider how the interactions change as the electron clouds and nuclear sheets cross over each other, see Figure A6.1.1.

Let x be the separation between the two nuclear sheets. If $x \geq d$ then the electron clouds do not overlap and the repulsive force between electron clouds (e-e force) and the repulsive force between the nuclear sheets (n-n force) will balance the attractive forces between electron clouds and nuclear sheets (e-n and n-e forces), leaving no net interaction, see Figure A6.1.1 left and Eq. (A6.1.1) top.

If $\frac{1}{2}d \leq x \leq d$ then the electron clouds overlap, by an amount $d - x$, but the electron clouds do not overlap the nuclei. The e-e force will be reduced by an amount proportional to the overlap squared, since the overlapping regions have no net interaction, see Figure A6.1.1 center and Eq. (A6.1.1) center.

If $0 \leq x \leq \frac{1}{2}d$ then the electron clouds overlap the nuclei, by an amount $\frac{1}{2}d - x$, but the nuclei do not cross over each other. The e-e force will be reduced as before but now the e-n and n-e forces will also be reduced, each by an amount proportional to twice the overlap, since the region of the electron cloud that is beyond the nuclei is now attracted to the nuclei in the opposite direction, see Figure A6.1.1 right and Eq. (A6.1.1) bottom.

To summarize, the force between the surfaces is

$$F \propto \begin{cases} 0 & \text{for} \quad x \geq d \\ -(d-x)^2 & \text{for} \quad \frac{1}{2}d \leq x \leq d \\ 2d(d-2x) - (d-x)^2 & \text{for} \quad 0 \leq x \leq \frac{1}{2}d \end{cases} \tag{A6.1.1}$$

and the corresponding potential is

$$\phi \propto \begin{cases} 0 & \text{for} \quad x \geq d \\ -\frac{1}{3}(d-x)^3 & \text{for} \quad \frac{1}{2}d \leq x \leq d \\ \frac{1}{2}d(d-2x)^2 - \frac{1}{3}(d-x)^3 & \text{for} \quad 0 \leq x \leq \frac{1}{2}d \end{cases} \tag{A6.1.2}$$

see Figure A6.1.2.

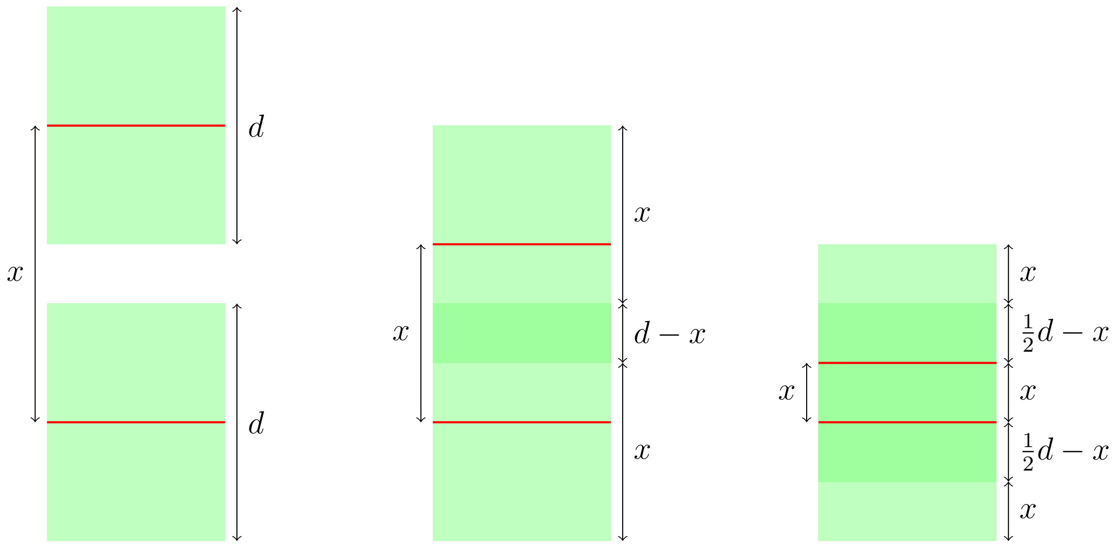


Figure A6.1.1: Left: no interaction ($x \geq d$); center: attraction ($\frac{1}{2}d \leq x \leq d$); right: attraction/repulsion ($0 \leq x \leq \frac{1}{2}d$).

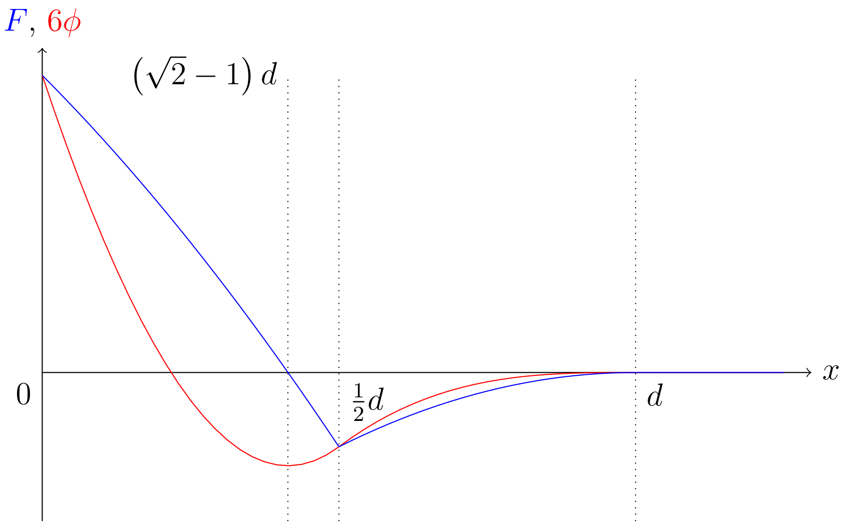


Figure A6.1.2: Force $F(x)$ and potential $\phi(x)$.