Homework 1 - Linear operators and eigenspaces

Q1.1. Show that if A is a linear operator then $AA^\dagger$ is Hermitian.

Q1.2. Show that the eigenvalues α of a Hermitian operator satisfy

$$\alpha^* = \alpha \tag{Q1.2.1}$$

Q1.3. Show that the eigenspaces of a Hermitian operator are orthogonal.

Q1.4. The linear operators A and B have commutator [1]

$$[A, B] = \gamma \qquad (\gamma \neq 0) \tag{Q1.4.1}$$

Show that they cannot have a common eigenvector.

[1] The right hand side of this equation is the linear operator $\gamma \times 1$, where $\gamma \in \mathbb{C}$ and 1 is the identity operator, but is just written as γ for simplicity.