

Homework 4 - Angular momentum

Q4.1. Let

$$\hat{L}_{ab} = \hat{x}_a \hat{p}_b - \hat{x}_b \hat{p}_a \quad (\text{Q4.1.1})$$

for $a, b \in \{1, 2, 3\}$, and

$$\hat{L}^2 = \hat{L}_{12}^2 + \hat{L}_{23}^2 + \hat{L}_{31}^2 \quad (\text{Q4.1.2})$$

Show that

$$[\hat{L}_{ab}, \hat{L}_{bc}] = i\hbar \hat{L}_{ca} \quad (\text{Q4.1.3})$$

and

$$[\hat{L}_{ab}, \hat{L}^2] = 0 \quad (\text{Q4.1.4})$$

for $a, b, c \in \{1, 2, 3\}$ and $a \neq b \neq c$.

Q4.2. Determine the eigenvalues of \hat{L}^2 and \hat{L}_{12} by considering the properties of

$$\hat{L}_{\pm} = \hat{L}_{23} \pm i\hat{L}_{31} \quad (\text{Q4.2.1})$$

in a manor similar to Section (1.2.4).