Homework 1

Q1.1. Starting from

$$z = re^{i\theta} \tag{Q1.1.1}$$

calculate

(a) $\dot{z}$

(b) $\ddot{z}$

and hence determine the radial and angular components of the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ in two dimensions.

Use PGF to draw a diagram illustrating your answer.

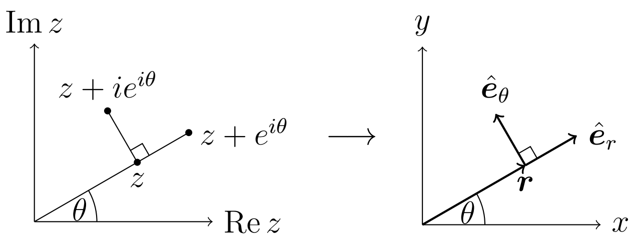
A1.1. (a) Differentiating Eq. (Q1.1.1) gives

$$\dot{z} = \left(\dot{r} + ir\dot{\theta}\right) e^{i\theta} \tag{A1.1.1}$$

(b) Differentiating Eq. (A1.1.1) gives

$$\ddot{z} = \left[\ddot{r} - r\dot{\theta}^2 + i\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\right] e^{i\theta} \tag{A1.1.2}$$

Converting from complex to vector representation, we have



(A1.1.3)

or equivalently

$$z \rightarrow \boldsymbol{r} \tag{A1.1.4}$$

$$e^{i\theta} \rightarrow \hat{\boldsymbol{e}}_r \tag{A1.1.5}$$

$$ie^{i\theta} = e^{i\left(\theta + \frac{\pi}{2}\right)} \rightarrow \hat{\boldsymbol{e}}_\theta \tag{A1.1.6}$$

Therefore Eqs. (A1.1.1) and (A1.1.2) become

$$\boldsymbol{v} = \dot{r}\hat{\boldsymbol{e}}_r + r\dot{\theta}\hat{\boldsymbol{e}}_\theta \tag{A1.1.7}$$

and

$$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\boldsymbol{e}}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{e}}_\theta \tag{A1.1.8}$$