Homework 6

Answers should be submitted, as both a tex file and a pdf file, to both me and the teaching assistants. You may use this file as a template.

Q6.1. Consider the set of functions satisfying the boundary condition

$$\phi(a) = \phi(b) = \phi_0 \tag{Q6.1.1}$$

Does it form a vector space?

Q6.2. Show that

(a)
$$A^{\dagger\dagger} = A \tag{Q6.2.1}$$

(b)
$$(AB)^\dagger = B^\dagger A^\dagger \tag{Q6.2.2}$$

(c)
$$\left(A^\dagger\right)^{-1} = \left(A^{-1}\right)^\dagger \tag{Q6.2.3}$$

Q6.3. Show that

(a) $AA^\dagger$ is Hermitian,

(b) e^{iH} is unitary,

(c) U_1U_2 is unitary.

where A is a linear operator, H is a Hermitian operator, and U_1 and U_2 are unitary operators.

Q6.4. Describe the properties of

$$\frac{|\phi\rangle \langle\psi|}{\langle\psi|\phi\rangle} \tag{Q6.4.1}$$