Homework 8

Answers should be submitted, as both a tex file and a pdf file, to both me and the teaching assistants. You may use this file as a template.

Q8.1. Show that

(a)
$$\delta(-x) = \delta(x) \tag{Q8.1.1}$$

(b)
$$\delta'(-x) = -\,\delta'(x) \tag{Q8.1.2}$$

(c)
$$x\,\delta(x) = 0 \tag{Q8.1.3}$$

(d)
$$x\,\delta'(x) = -\,\delta(x) \tag{Q8.1.4}$$

Q8.2. Show that

(a)
$$\delta(x) = \theta'(x) \tag{Q8.2.1}$$

where
$$\theta(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x > 0 \end{cases} \tag{Q8.2.2}$$

(b)
$$\int_{-\infty}^{\infty} dy\,\delta'(x-y)\,f(y) = f'(x) \tag{Q8.2.3}$$

Q8.3. Use PGF to draw diagrams illustrating $\delta(x)$ and $\delta'(x)$.

Q8.4. Show that

(a)
$$\delta(x) = \lim_{\epsilon\to 0^+} \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\, e^{ikx-\epsilon k^2} \tag{Q8.4.1}$$

(b)
$$\delta(x) = \lim_{\Lambda\to\infty} \frac{1}{2\pi} \int_{-\Lambda}^{\Lambda} dk\, e^{ikx} \tag{Q8.4.2}$$