Homework 2 - Tensor algebra

Q2.1. Use diagrammatic methods to show that

(a)

$$\vec{v} \cdot (\underline{\omega} \wedge \underline{\sigma}) = (\vec{v} \cdot \underline{\omega})\, \underline{\sigma} - (\vec{v} \cdot \underline{\sigma})\, \underline{\omega} \tag{Q2.1.1}$$

(b)

$$\vec{\vec{v}} \cdot (\underline{\omega} \wedge \underline{\sigma}) = (\vec{\vec{v}} \cdot \underline{\sigma}) \cdot \underline{\omega} \tag{Q2.1.2}$$

A2.1. (a) Rescaling

$$\underline{\hat{\omega}} = \frac{\underline{\omega}}{\vec{v} \cdot \underline{\omega}} \tag{A2.1.1}$$

$$\underline{\hat{\sigma}} = \frac{\underline{\sigma}}{\vec{v} \cdot \underline{\sigma}} \tag{A2.1.2}$$

then Figure A2.1.1 gives

$$\vec{v} \cdot (\underline{\hat{\omega}} \wedge \underline{\hat{\sigma}}) = \underline{\hat{\sigma}} - \underline{\hat{\omega}} \tag{A2.1.3}$$

and hence Eq. (Q2.1.1).

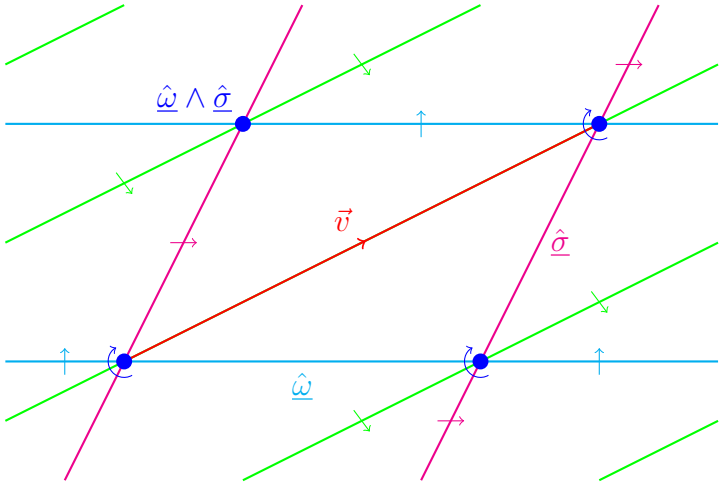

Figure A2.1.1: $[\vec{v} \cdot (\underline{\hat{\omega}} \wedge \underline{\hat{\sigma}})] = (\underline{\hat{\sigma}} - \underline{\hat{\omega}})$

(b) Rescaling

$$\vec{\hat{v}} = \frac{\vec{\vec{v}}}{\vec{\vec{v}} \cdot (\underline{\omega} \wedge \underline{\sigma})} \tag{A2.1.4}$$

then Figure A2.1.2 gives

$$\vec{\hat{v}} \cdot (\underline{\omega} \wedge \underline{\sigma}) = 1 = (\vec{\vec{v}} \cdot \underline{\sigma}) \cdot \underline{\omega} \tag{A2.1.5}$$

and hence Eq. (Q2.1.2).

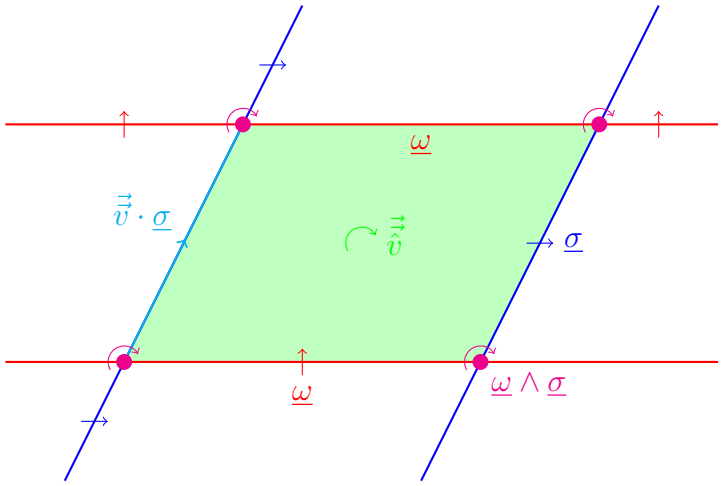

Figure A2.1.2: $\vec{\hat{v}} \cdot (\underline{\omega} \wedge \underline{\sigma}) = (\vec{\hat{v}} \cdot \underline{\sigma}) \cdot \underline{\omega}$

Q2.2. Using Eq. (1.1.22), show that

(a)

$$(\underline{\omega} \wedge \underline{\sigma}) \cdot (\vec{u} \wedge \vec{v}) = (\underline{\omega} \cdot \vec{u})\,(\underline{\sigma} \cdot \vec{v}) - (\underline{\omega} \cdot \vec{v})\,(\underline{\sigma} \cdot \vec{u}) \tag{Q2.2.1}$$

(b)

$$\underline{\sigma} \cdot (\vec{u} \wedge \vec{v} \wedge \vec{w}) = (\underline{\sigma} \cdot \vec{u})\,\vec{v} \wedge \vec{w} + (\underline{\sigma} \cdot \vec{v})\,\vec{w} \wedge \vec{u} + (\underline{\sigma} \cdot \vec{w})\,\vec{u} \wedge \vec{v} \tag{Q2.2.2}$$

A2.2. (a)

$$\begin{aligned}
(\underline{\omega} \wedge \underline{\sigma}) \cdot (\vec{u} \wedge \vec{v}) &= [(\underline{\omega} \wedge \underline{\sigma}) \cdot \vec{v}] \cdot \vec{u} && \text{(A2.2.1)}\\
&= [\underline{\omega}\,(\underline{\sigma} \cdot \vec{v}) - (\underline{\omega} \cdot \vec{v})\,\underline{\sigma}] \cdot \vec{u} && \text{(A2.2.2)}\\
&= (\underline{\omega} \cdot \vec{u})\,(\underline{\sigma} \cdot \vec{v}) - (\underline{\omega} \cdot \vec{v})\,(\underline{\sigma} \cdot \vec{u}) && \text{(A2.2.3)}
\end{aligned}$$

(b)

$$\begin{aligned}
\underline{\sigma} \cdot (\vec{u} \wedge \vec{v} \wedge \vec{w}) &= (\underline{\sigma} \cdot \vec{u})\,\vec{v} \wedge \vec{w} + [\underline{\sigma} \cdot (\vec{v} \wedge \vec{w})] \wedge \vec{u} && \text{(A2.2.4)}\\
&= (\underline{\sigma} \cdot \vec{u})\,\vec{v} \wedge \vec{w} + [(\underline{\sigma} \cdot \vec{v})\,\vec{w} - (\underline{\sigma} \cdot \vec{w})\,\vec{v}] \wedge \vec{u} && \text{(A2.2.5)}\\
&= (\underline{\sigma} \cdot \vec{u})\,\vec{v} \wedge \vec{w} + (\underline{\sigma} \cdot \vec{v})\,\vec{w} \wedge \vec{u} + (\underline{\sigma} \cdot \vec{w})\,\vec{u} \wedge \vec{v} && \text{(A2.2.6)}
\end{aligned}$$