Homework 7 - Abstract index notation

Q7.1. Express Eqs (1.4.3), (1.4.6) and (1.4.8) in abstract index notation.

Q7.2. Express Eqs. (1.5.2) and (1.5.3) in abstract index notation and show that

(a)
$$v^\alpha = e^\alpha_{\mathbf{a}} v^{\mathbf{a}} \tag{Q7.2.1}$$

(b)
$$\omega_{\mathbf{a}} v^{\mathbf{a}} = \omega_\alpha v^\alpha \tag{Q7.2.2}$$

(c)
$$e^\alpha_{\mathbf{a}} e^{\mathbf{b}}_\alpha = \delta^{\mathbf{b}}_{\mathbf{a}} \tag{Q7.2.3}$$

explaining the meaning of all terms.

Q7.3. Using abstract index notation, show that

(a)
$$\vec{v} \cdot \left(\underline{\omega} \wedge \underline{\underline{\sigma}}\right) = \left(\vec{v} \cdot \underline{\omega}\right) \underline{\underline{\sigma}} + \left(\vec{v} \cdot \underline{\underline{\sigma}}\right) \wedge \underline{\omega} \tag{Q7.3.1}$$

(b)
$$\vec{\vec{v}} \cdot \left(\underline{\omega} \wedge \underline{\underline{\sigma}}\right) = \left(\underline{\omega} \cdot \vec{\vec{v}}\right) \cdot \underline{\underline{\sigma}} + \left(\vec{\vec{v}} \cdot \underline{\underline{\sigma}}\right) \underline{\omega} \tag{Q7.3.2}$$